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Correspondences of the semigroup of endomorphisms of an equivalence relation

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Let ρ be a binary relation on a set X, $End(\rho)$ be a set of all endomorphisms of the relation ρ. An ordered pair (φ, ψ) of transformations φ and ψ of the set X is called an endotopism [1] if $(x, y) \in \rho$ implies $(x\varphi, y\psi) \in \rho$ for all $x, y \in X$. The set of all endotopisms of ρ is a semigroup relative to the operation of the direct product of transformations. This semigroup is called the semigroup of endotopisms of the relation ρ and it is denoted by $Et(\rho)$.

Let G be a universal algebra. If we consider a subalgebra of $G \times G$ as a binary relation on G, then the set $S(G)$ of all subalgebras of $G \times G$ is a semigroup relative to the operation of the composition of binary relations. Elements of this semigroup are called correspondences of the algebra G [2].

Lemma 1. *For any equivalence relation α on a set X the semigroup $Et(\alpha)$ is the correspondence of the semigroup $End(\alpha)$.*

Let α be an arbitrary equivalence relation on the set X. We define a small category K such that $Ob\,K = X/\alpha$ and $Mor(A, B)$ is the set of all mappings from A to B for all $A, B \in ObK$.

We designate by W the wreath product $\mathfrak{J}(X/\alpha)wrK$ of the symmetric semigroup $\mathfrak{J}(X/\alpha)$ with the small category K (see, e.g., [3]) and by K^2_* the full subcategory of category K^2 defined on $ObK^2_* = \{(A; A) | A \in ObK\}$.

Theorem 1. *Let α be an equivalence on a set X, K be the small category defined above. The correspondence $Et(\alpha)$ of the semigroup $End(\alpha)$ can be exactly represented as:*

1) the subdirect product of the monoid $(\Im\,(X/\alpha)\,wrK) \times (\Im\,(X/\alpha)\,wrK)$;
2) the wreath product $\Im\,(X/\alpha)\,wrK^2_*$ of $\Im\,(X/\alpha)$ with the small category K^2_*.

Corollary 1. *For any equivalence relation α on a finite set X we have*

$$|Et(\alpha)| = \sum_{\varphi \in \Im(X/\alpha)} \Big(\prod_{A \in X/\alpha} |A\varphi|^{|A|} \Big)^2.$$

In addition, we study such correspondences of the semigroup of all endomorphisms of an equivalence relation as the monoid of all strong endotopisms and the group of all autotopisms of the given equivalence relation.

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