

# The 9-th International Algebraic Conference in Ukraine

L'viv, Ukraine, July 8–13, 2013

## Abstracts of Reports

<i>V. I. Murashka</i> , On $R$ -subnormal subgroups . . . . .	133
<i>O. Mykitysey</i> , Continuity of symmetric products of capacities . . . . .	134
<i>O. Mylyo</i> , On zeros of polynomial of the elliptic functions . . . . .	135
<i>E. Myslovets</i> , On formation of finite $ca$ - $\mathfrak{F}$ -groups . . . . .	136
<i>M.I. Naumik</i> , About semigroups of linear relations . . . . .	137
<i>M. Nazarov</i> , On application of self-induced metric on groupoids . . . . .	138
<i>B. Novikov and G. Zholtkevych</i> , On generalized automata . . . . .	139
<i>O. Nykyforchyn, D. Repovš</i> , On natural liftings of functors in the category of compacta to categories of topological algebra and analysis . . . . .	140
<i>A. Oliynyk</i> , Free products and infinite unitriangular matrices . . . . .	141
<i>B. Oliynyk</i> , The normal structure of the diagonal limit of hyperoctahedral groups with doubling embeddings . . . . .	142
<i>R.M. Oliynyk</i> , On the lattice of quasi-filters of left congruence on a Clifford semigroups . . . . .	143
<i>I.A. Palas</i> , Prime abelian groups in a localic topos and test of injectivity . . .	144
<i>O.V. Petrenko, I.V. Protasov, S. Slobodianiuk</i> , Asymptotic structures of cardinals . . . . .	145
<i>M. Plakhotnyk</i> , The chain length in POset and its connection with solution of linear equations over $\mathbb{Z}$ . . . . .	146
<i>R. Popovych</i> , Sharpening of explicit lower bounds on elements order for finite field extensions $F_q[x]/\Phi_r(x)$ . . . . .	147
<i>M. Pratsiovytyi, N. Vasylenko</i> , Using a variety of images of real numbers for modeling and analysis of fractal properties of nowhere differentiable functions . . . . .	148
<i>V.M. Prokip</i> , Normal form with respect to similarity of involutory matrices over a principal ideal domain . . . . .	149
<i>I.V. Protasov, S. Slobodianiuk</i> , Ultracompanion's characterization of subsets of a group . . . . .	150
<i>O. Prianychnykova</i> , Algebras of Language Transformations in Labelled Graphs	151
<i>N.M. Pyrch</i> , Absolute $G$ -retracts and their applications . . . . .	152
<i>A. S. Radova</i> , Divisor function $\tau_3(w)$ in arithmetic progression . . . . .	153
<i>I.Iu. Raievska, M.Iu. Raievska, Ya.P. Sysak</i> , Finite local nearrings with multiplicative Miller-Moreno group . . . . .	154
<i>E. A. Romanenko</i> , Correspondences of the semigroup of endomorphisms of an equivalence relation . . . . .	155
<i>A.M. Romaniv</i> , Greatest common left divisor of matrices one of which is disappear . . . . .	156
<i>N.M. Rusin</i> , 2-state ZC-automata generating free groups . . . . .	157
<i>A.V. Russyev</i> , Conjugacy in finite state wreath powers of finite permutation groups . . . . .	158
<i>Yu.M. Ryabuchin, L.M. Ryabuchina</i> , Radical subrings in the field of $Q$ of rational numbers . . . . .	159
<i>O. Ryabukho</i> , Zero divisors in the semigroups of infinite dimensional special triangular matrices . . . . .	160
<i>A.V. Sadovnichenko</i> , Linear groups with a spacious family of $G$ -invariant subspaces . . . . .	161

# Correspondences of the semigroup of endomorphisms of an equivalence relation

E. A. Romanenko

Luhansk National Taras Shevchenko University, Luhansk, Ukraine

e.a.rom@mail.ru

Let  $\rho$  be a binary relation on a set  $X$ ,  $End(\rho)$  be a set of all endomorphisms of the relation  $\rho$ . An ordered pair  $(\varphi, \psi)$  of transformations  $\varphi$  and  $\psi$  of the set  $X$  is called an endotopism [1] if  $(x, y) \in \rho$  implies  $(x\varphi, y\psi) \in \rho$  for all  $x, y \in X$ . The set of all endotopisms of  $\rho$  is a semigroup relative to the operation of the direct product of transformations. This semigroup is called the semigroup of endotopisms of the relation  $\rho$  and it is denoted by  $Et(\rho)$ .

Let  $G$  be a universal algebra. If we consider a subalgebra of  $G \times G$  as a binary relation on  $G$ , then the set  $S(G)$  of all subalgebras of  $G \times G$  is a semigroup relative to the operation of the composition of binary relations. Elements of this semigroup are called correspondences of the algebra  $G$  [2].

**Lemma 1.** *For any equivalence relation  $\alpha$  on a set  $X$  the semigroup  $Et(\alpha)$  is the correspondence of the semigroup  $End(\alpha)$ .*

Let  $\alpha$  be an arbitrary equivalence relation on the set  $X$ . We define a small category  $K$  such that  $Ob K = X/\alpha$  and  $Mor(A, B)$  is the set of all mappings from  $A$  to  $B$  for all  $A, B \in Ob K$ .

We designate by  $W$  the wreath product  $\mathfrak{S}(X/\alpha)wrK$  of the symmetric semigroup  $\mathfrak{S}(X/\alpha)$  with the small category  $K$  (see, e.g., [3]) and by  $K_*^2$  the full subcategory of category  $K^2$  defined on  $Ob K_*^2 = \{(A; A) | A \in Ob K\}$ .

**Theorem 1.** *Let  $\alpha$  be an equivalence on a set  $X$ ,  $K$  be the small category defined above. The correspondence  $Et(\alpha)$  of the semigroup  $End(\alpha)$  can be exactly represented as:*

- 1) the subdirect product of the monoid  $(\mathfrak{S}(X/\alpha)wrK) \times (\mathfrak{S}(X/\alpha)wrK)$ ;
- 2) the wreath product  $\mathfrak{S}(X/\alpha)wrK_*^2$  of  $\mathfrak{S}(X/\alpha)$  with the small category  $K_*^2$ .

**Corollary 1.** *For any equivalence relation  $\alpha$  on a finite set  $X$  we have*

$$|Et(\alpha)| = \sum_{\varphi \in \mathfrak{S}(X/\alpha)} \left( \prod_{A \in X/\alpha} |A\varphi|^{|A|} \right)^2.$$

In addition, we study such correspondences of the semigroup of all endomorphisms of an equivalence relation as the monoid of all strong endotopisms and the group of all autotopisms of the given equivalence relation.

1. Popov B. V. Endotopism semigroups of an  $\mu$ -ary relation, Uch. zap. LGPI im. A.I. Gercen, v. 274 (1965), 184-201 (In Russian).
2. Kurosh A. G., General algebra (lectures 1969-70 school year). – M.: Nauka, 1974, P. 110 (In Russian).
3. Knauer U., Nieporte M. Endomorphisms of graphs. I. The monoid of strong endomorphisms. Arch. Math., Vol. 52 (1989), 607 – 614.