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MESHING CHARACTERISTICS OF PROFILE SHIFTED CYLINDRICAL QUASI-INVOLUTE ARC-TOOTH-TRACE GEARS. PART 2. CALCULATION RESULTS

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Abstract: Based on the results obtained in the previous research, the values of meshing characteristics of profile shifted quasi-involute arc-tooth-trace gears is calculated. The influence of the values of profile shift coefficient and the angle of tooth trace on meshing characteristic distribution on the tooth flank surface along tooth trace is defined. The results can be used for design of profile shifted quasi-involute arc-tooth-trace gears, cut by Gleason-type cutters with different profile angle value.

KEYWORDS: arc-tooth-trace gears, profile shift coefficient, sliding velocity, rolling velocity of mating teeth surfaces, reduced curvature, specific sliding, angle between sliding velocity vector and contact line

1 Introduction

Gear pairs are the crucial part of any machine or system, ranging from airplanes to home appliances. According to [1], the improvement of gears quality opens the way to sustainability in gear manufacturing. Moreover, the characteristics of gears usually determine machine's durability, and they can directly impact the operational characteristics that is explicitly expressed in hydraulics machines [2, 3]. Parallel-shaft gears are the most extensively used in the transmission systems. This is why the technologies for cylindrical gear manufacturing has the largest market share, and the market of this technologies "is expected to grow with the highest CAGR" [4]. This indicates a growing demand for the mentioned type of gears.

Spur and helical (including double helical and herringbone [5]) gears are the most common types of parallel-shaft gears. Now the arc-tooth-trace (ATT) gears are a small part of parallel-shaft gears, but they have advantages compared to spur and helical gears, such as higher loading capacity and service life, smoothness of operation due to the higher overlap ratio value, lower

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size and weight, absence of the axial force when in operation. This is why it can be concluded that the improvement of cylindrical ATT gears is a relevant problem in the present time.

2 Literature review

The increasing of gear's quality is a relevant problem since the time when the gars exist, and arc-tooth-trace gears are not the exception. The path from spur gears to ATT cylindrical ones was made by the industry and science through the development of helical and double helical or herringbone gears. They are under the focus of researchers nowadays. Their research and development is supported by advanced methods of manufacturing, in particular, by using the power skiving process [6, 7]. The study of the latest articles shows that the majority of helical and double helical gears have the involute teeth profile. The most common directions of their research are as follows: more reliable assessment of the vibration characteristics [8] research of dynamics [9], in particular, caused by failures [10], manufacturing errors [11, 12], lubrication and wear [13, 14]. The most common way of their improvement is the modification of the teeth geometry. Although the authors of [13] and [14] focused their research on lubrication and wear problem, they considered gear pairs with profile [13] and axial [14] modification. Zou et al. [15] developed an improved algorithm of the teeth surface topological modification that allowed to reduce the maximum contact stresses by 28.04% compared with the unmodified gear pairs due to more uniform load share between the teeth. The dynamic characteristics of the herringbone gearing also were improved. The authors of [15] in the article [16] proposed an optimal 3d modification technology which include both profile and axial tooth modification. Such a modification provides the reduction of vibration and noise. The 3d modification technology was combined with the contact and dynamic characteristics in one research that allowed authors to optimise meshing characteristics of herringbone gear pairs and obtain the optimal modification which can make the rotation of the system smoother. Wang et al. [17] considered the highly loaded gear pair which has the significant deviations from theoretical contact due to the errors and deformations of teeth under load, especially on mesh-in and mesh-out points. The authors proposed the teeth surface modification and used their own optimal analysis method to evaluate the effect of modification on the contact pressure, thermoelastic deformation of the teeth, vibration and noise. According to the conclusions of the authors of [17], the optimal teeth surface modification is able to improve all the above mentioned characteristics. Lee et al. [18] simulated the meshing of a pair of herringbone double circular-arc helical gears, generated by the basic rack with the parameters that correspond to GB/T12759-1991 standard. Based on the simulation results the authors of [18] proposed to apply the longitudinal crowning technology for such type of gearing.

As it was mentioned above, ATT gears have become the next step in the development of highly loaded cylindrical gears. On the early stage of their development only the quasi-involute gears with circular-arc tooth trace were considered. They are called "quasi-involute" in some research, because the involute meshing takes place only in the middle section of the arc. The study of the latest ATT gears research shows that the different types of curves are applicable to outline the tooth trace. Among them, circle, cycloid and hyperbola are the most widely used. The different teeth and basic rack profiles are applicable for ATT gears. The most common, besides the involute one, are the circle-arc and cycloidal profiles and their combinations. The illustrative example of combined profile is presented in [19]. The tooth trace is a segment of a circular arc, while the tooth profile is divided by pitch circle into two parts: a standard arc segment and a segment of cycloid. According to the simulation results the contact and bending stresses are reduced in comparison with the involute gear drive. Another example of ATT gears with complex geometry is cylindrical gear with variable hyperbolic circular-arc-tooth-trace (VH-CATT) [20, 21]. In [20] Wei et al. presented the contact stress analysis of VH-CATT gears using their own formula and FEA. It shows that the formula has a high accuracy. In [21] the same authors investigated the wear of VH-CATT gears using their own wear prediction model. They have established that the wear of teeth is reduced by 23.16% compared to the wear of spur gears. In [22] the mathematical models of ATT gears with both circular and cycloidal tooth trace were obtained. A circular arc was also used to outline the profile of the basic rack. It was shown that the above mentioned ATT gears have improved all meshing characteristics compared to the characteristics of quasi-involute gears. Despite the significant research interest to more complex ATT gears [19-22] their implementation is limited by their sensitiveness to the errors and deformations under load. This is why the research of the quasi-involute gears with circular-arc tooth trace continues. Zhijun et al. [23] investigated the influence of geometrical parameters on the contact strength of ATT gear, which was compared to the helical gear with the same line of contact. It was established that in order to increase the contact strength of ATT gear the ratio of Gleason-type cutter's radius to gear width should be decreased. Zhang et al. [24] proposed a planetary gear train as the processing device combined with rack-type cutter which has multiple blades. The condition of meshing of the teeth generated using the device were investigated using the meshing theory and the graphical processing. It was shown that: (i) the motion of cutter realized by the device can form an ideal tooth profile; (ii) the more blades will be installed on the cutter – the higher precision will be obtained. All the above mentioned research does not take into account profile shifting. In [25] the geometry of ATT gears at profile shift was described, but functional relationships for determining the geometric-kinematic meshing characteristics depending on parameters of the generating surface was not obtained. In the first part of this research [26] geometric-kinematic meshing characteristics and the conditions of teeth undercutting and tip pointing for profile shifted quasi-involute ATT gears were obtained. It was found, as it was expected, that the most undesirable values of geometric-kinematic meshing characteristics take place in the central section of the tooth while the most dangerous in terms of the teeth top land pointing are the peripheral transverse sections. But real distribution of geometric-kinematic meshing characteristics values was not obtained.

Thus, the objective of this research is to analyze the values of meshing characteristics described in [26] for profile shifted quasi-involute ATT gears and define the influence of profile shift coefficient x on meshing characteristics along the tooth trace line at different values of tooth trace angle β.

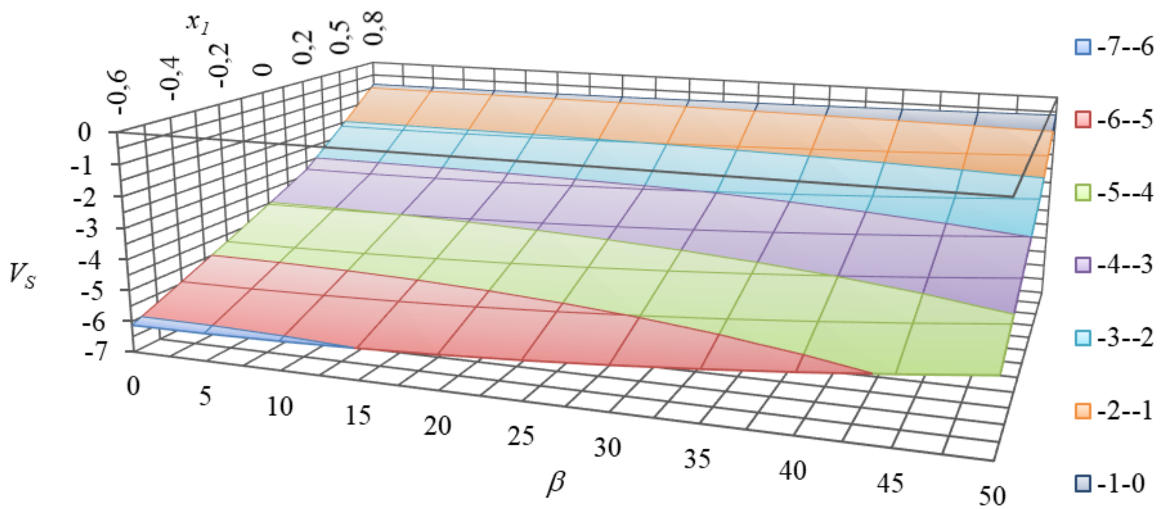

Fig. 1 Sliding velocity

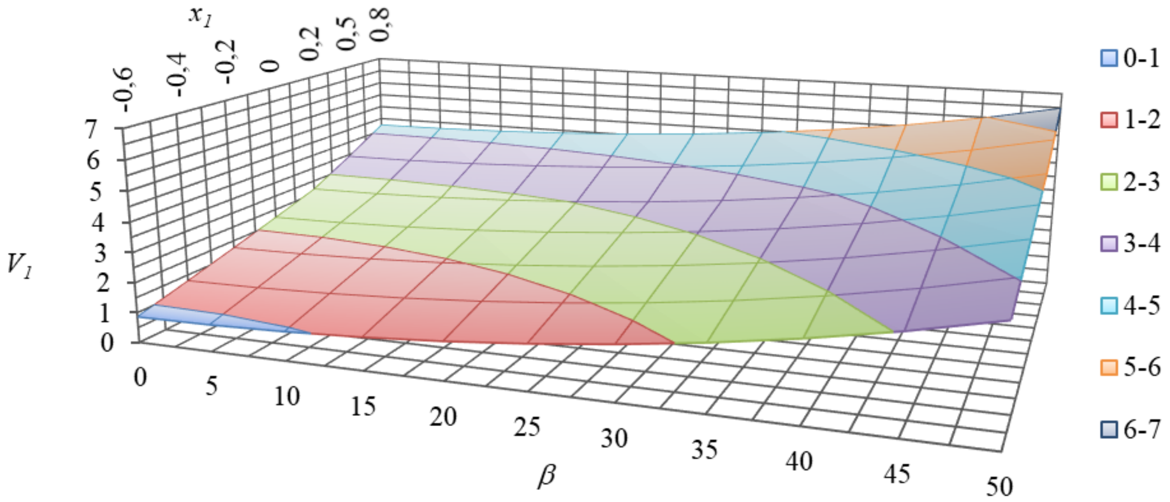

Fig. 2 Rolling velocity on pinion tooth

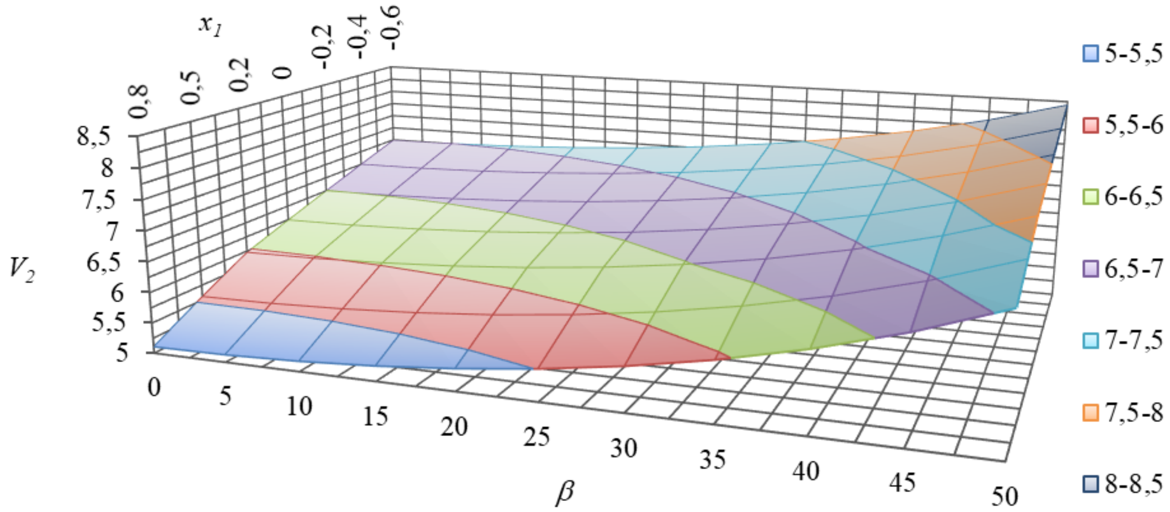

Fig. 3 Rolling velocity on gear tooth

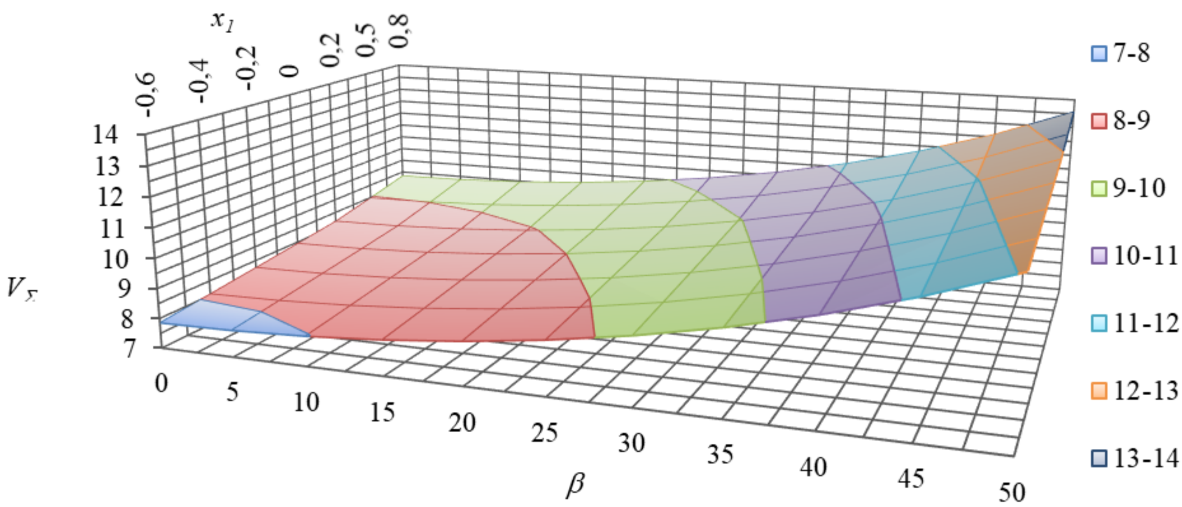

Fig. 4 Total rolling velocity

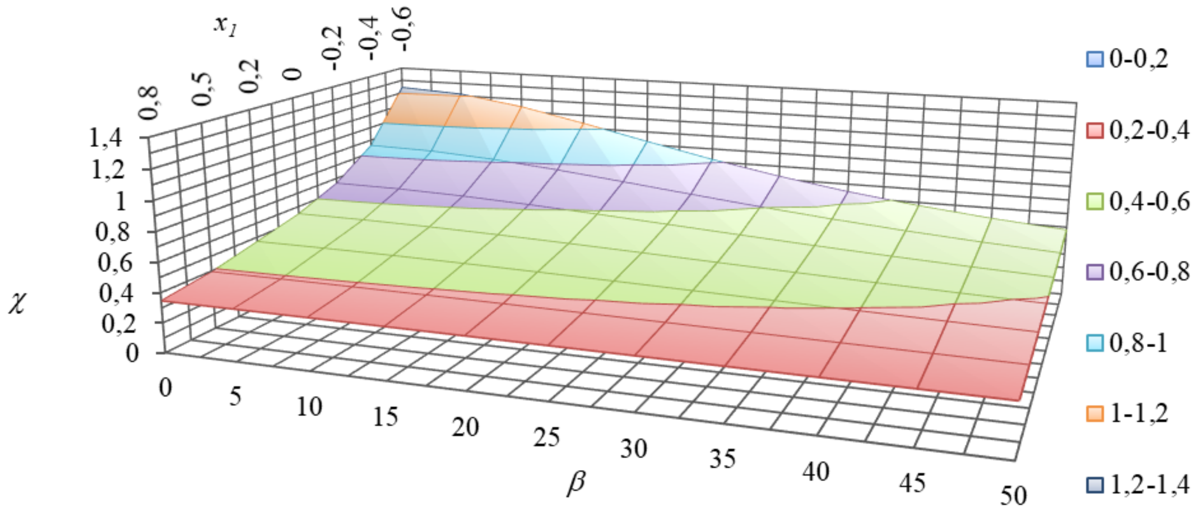

Fig. 5 Relative curvature

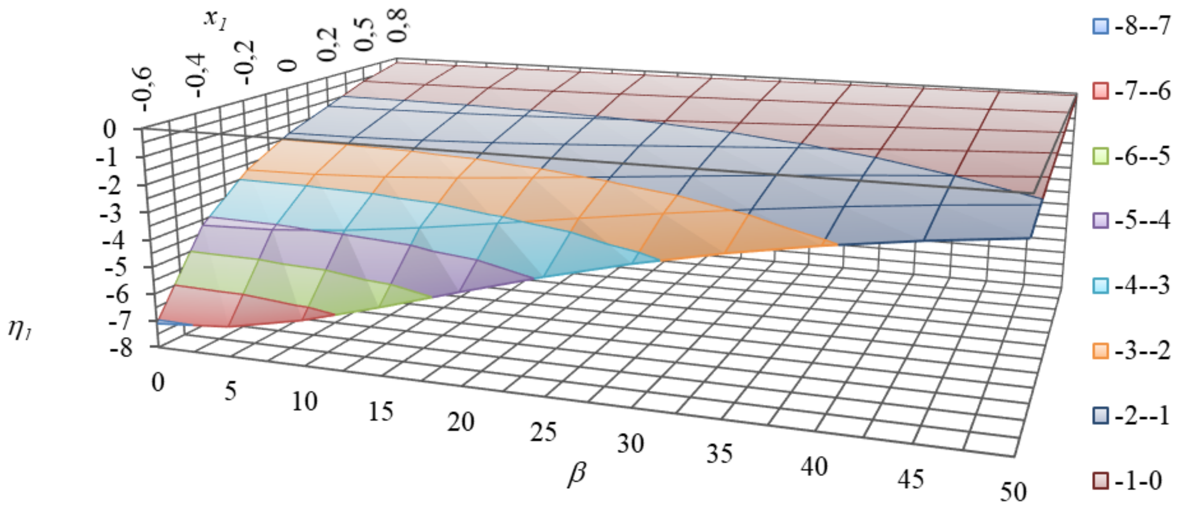

Fig. 6 Specific sliding coefficient on pinion tooth

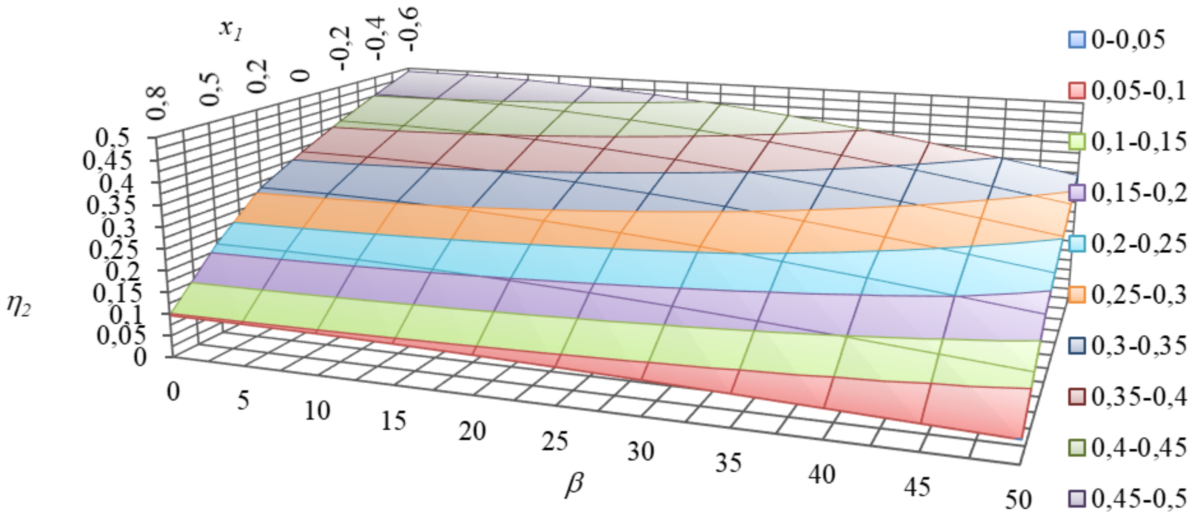

Fig. 7 Specific sliding coefficient on gear tooth

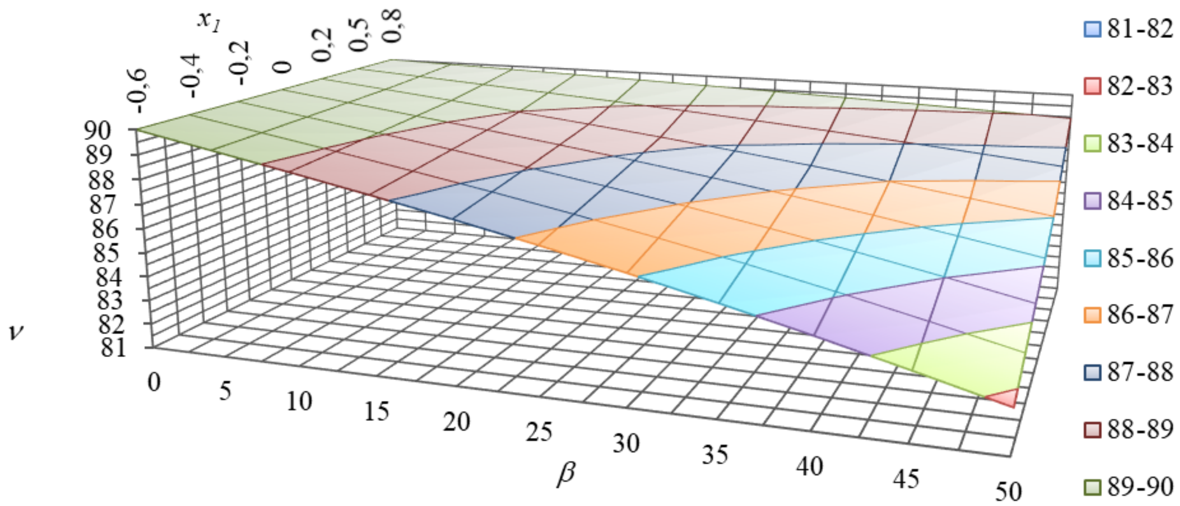

Fig. 8 Angle between sliding velocity vector and contact line

3 Meshing characteristics calculation

Let us calculate the geometric-kinematic meshing characteristics using the formulas obtained in the first part of this research [26]. In order to illustrate the influence of the profile shift coefficient on quasi-involute ATT gears meshing characteristics Figs. 1-8 are drawn.

In order to illustrate the effect of both positive and negative profile shift on the meshing characteristics the calculation was done taking into account the parameters of gearing which are as follows: the value of pressure angle is conventional and the most common $\alpha = 20°$; the numbers of teeth chosen are $z_1 = 28$, $z_2 = 50$, taking into account the recommendations of [27] to avoid undercutting and tip pointing at the range of profile shift coefficient of the pinion $x_1 = -0.6 ... +0.8$; the mating gears under consideration fulfill the condition $x_1 + x_2 = 0$, where x_2 is profile shift coefficient of the gear; the nominal radius of Gleason-type cutting tool is $R_C = 10$ (related to module $m = 1$ mm). All the meshing characteristics are calculated in the gear-in point of contact when the lowest point of the pinion profile contacts with the top point of the gear profile. In this point the characteristics of contact are usually the most unfavorable, therefore the meshing characteristics analysis is of interest in this particular case. The entire calculation was done using the formulas from [26] as follows: Eq. 7 for V_S value; Eqs. 8 for V_1 and V_2 values; Eq. 12 for V_Σ value; Eq. 16 for χ value; Eqs. 19 for η_1 and η_2 values; Eq. 20 for ν value. All the characteristics on Figs. 1-8 are related to module $m = 1$ mm, the values of V_S, V_1, V_2 and V_Σ are calculated at the angular velocity of the pinion $\omega_1 = 1s^{-1}$.

4 Results and discussions

The analysis of Figs. 1-8 shows that the presented results are the same as the results of the preliminary analysis carried out in [26], i.e. when the value of β angle increases: the absolute value of sliding velocity V_S decreases; the values of rolling velocities V_1, V_2, and V_Σ increase; the value of relative curvature χ decreases, the absolute values of specific sliding η_1 and η_2 decrease; the value of ν angle also decreases. It should be noted that the decreasing of χ value and η_1 absolute value is the most noticeable at the extreme negative shifting, i.e. at $x_1 = -0.6$, while their decreasing at $x_1 = +0.8$ is insignificant. This is due to the fact that the gear under consideration, i.e. with $z_1 = 28$, is closer to undercutting at $x_1 = -0.6$ compared to the one at

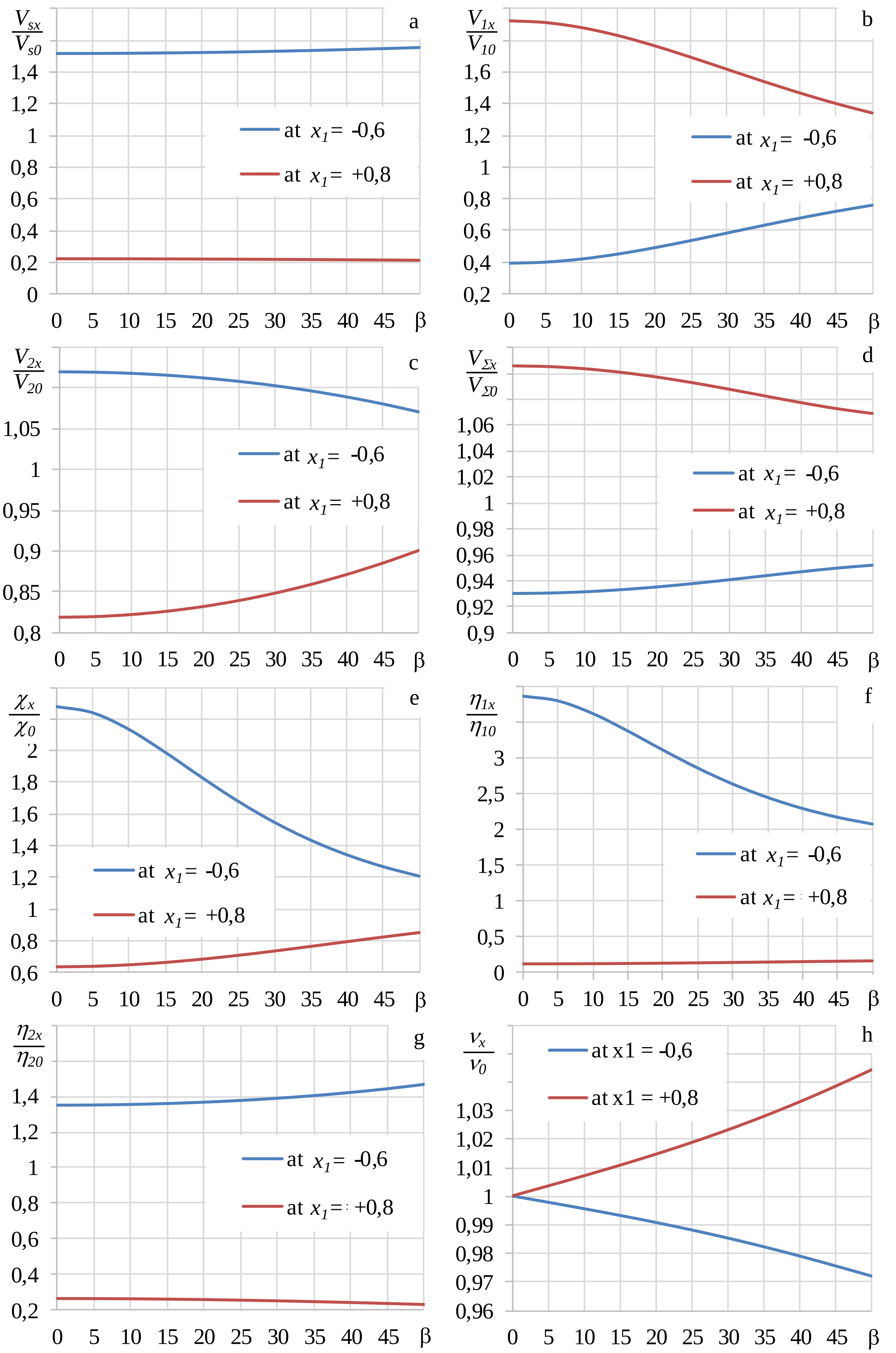

Fig. 9 Ratios of the characteristics values at $x = -0.6$ and $x = +0.8$ with the ones at $x = 0$

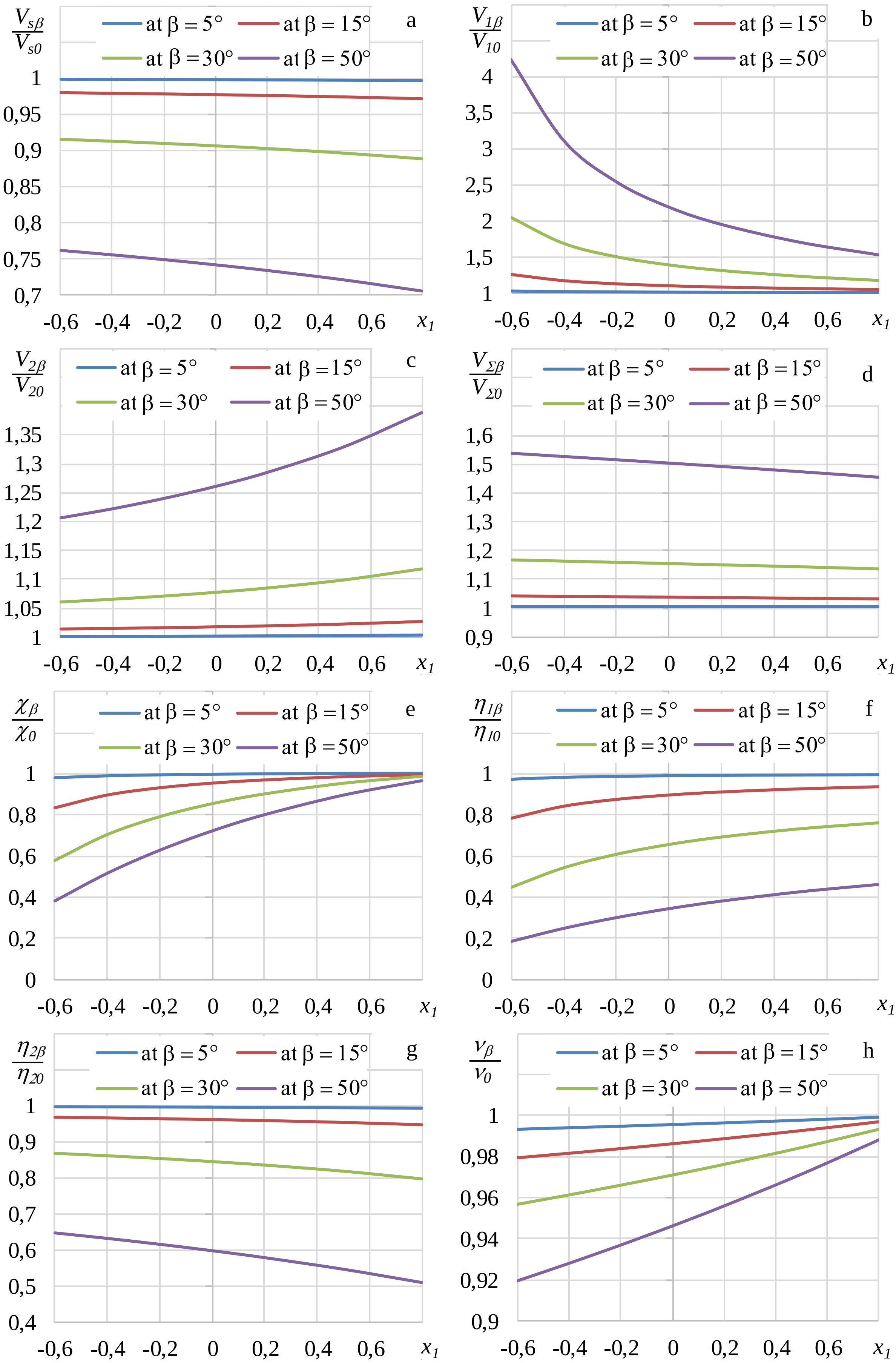

Fig. 10 Ratios of the characteristics values at $\beta = 5°...50°$ with the ones at $\beta = 0°$

$x_1 = +0.8$. The quantitative assay of meshing characteristics was carried out in two ways: (i) when the value of x coefficient is hold constant (ii) when the value of β angle is hold constant. In both cases the value of characteristic under influence of x or β was divided by this characteristic without the influence, i.e. at $x_1 = x_2 = 0$ or $\beta = 0°$. The results of above mentioned ratio calculations for both cases are presented in Fig. 9 and Fig 10 respectively.

In Figs. 9 and 10 the meshing characteristics are marked as follows: sliding velocity (a); rolling velocity on the pinion tooth (b); rolling velocity on the gear tooth (c); total rolling velocity (d); relative curvature (e); specific sliding on the pinion tooth (f); specific sliding on the gear tooth (g); angle between sliding velocity vector and contact line (h).

The results presented in Fig.9a show that V_S does not depend on β angle at positive shift on the pinion ($V_{S(+0.8)}/V_{S(0)} \approx 0.21$), and the increasing of $V_{S(-0.6)}/V_{S(0)}$ at negative shift on the pinion is insignificant. The influence of β angle on η_2 is almost the same (Fig. 9g). The ratio of V_1 at $x_1 = +0.8$ to the one at $x_1 = x_2 = 0$ ($V_{1(+0.8)}/V_{1(0)}$) reaches its maximum value in the narrow range of β angle at $\beta = 0°...15°$, while at $\beta = 15°...50°$ the ratio decreases significantly, tending to 1 (Fig. 9b). At $x_1 = -0.6$ the ratio $V_{1(-0.6)}/V_{1(0)}$ reaches its minimum value similarly at $\beta = 0°...15°$, and the ratio increases significantly at $\beta = 15°...50°$, tending to 1 (Fig. 9b). The influence of β angle on V_Σ is the same in the ranges of $\beta = 0°...15°$ and $\beta = 15°...50°$, but the ratio values are lower (Fig. 9d). The ratios of V_2 at $x_1 = +0.8$ and $x_1 = -0.6$ to the one at $x_1 = x_2 = 0$ ($V_{2(+0.8)}/V_{2(0)}$ and $V_{2(-0.6)}/V_{2(0)}$ respectively) change their values in a similar way at $\beta = 0°...15°$ and $\beta = 15°...50°$, but $V_{2(+0.8)}/V_{2(0)}$ increases while $V_{2(-0.6)}/V_{2(0)}$ decreases (Fig. 9c). The ratio of $\chi_{(+0.8)}/\chi_{(0)}$ at $x_1 = +0.8$ does not almost depend on β angle, while the ratio $\chi_{(-0.6)}/\chi_{(0)}$ at $x_1 = -0.6$ decreases (Fig. 9e). Two features of the ratio decreasing should be mentioned: the ratio reaches its maximum value at $\beta = 0°$; at $\beta = 0°...5°$ the ratio $\chi_{(-0.6)}/\chi_{(0)}$ is outlined by a gently sloping curve; at $\beta = 5°...50°$ the curve of $\chi_{(-0.6)}/\chi_{(0)}$ is steep (Fig. 9e). The influence of β angle on η_1 is the same (Fig. 9f). The ratio of ν at positive ($\nu_{(+0.8)}/\nu_{(0)}$) and negative ($\nu_{(-0.6)}/\nu_{(0)}$) profile shift on the pinion are almost in direct and inverse proportion respectively to β angle (Fig. 9h).

The effect of profile shifting on the meshing characteristics of spur and helical gears at $\beta = const$ is well understood, but not in case of ATT gears. This is why the influence of x coefficient is defined in connection with different values of β angle (Fig 10). The results presented in Fig. 10a show that for $\beta = 0°...15°$ the ratio $V_{S\beta}/V_{S0}$ is almost constant and very close to 1 in the range of $x_1 = -0.6...+0.8$. The influence of x coefficient increases in the transverse cross sections that are distant from the middle-arc section, i.e. at $\beta = 15°...50°$. Moreover, at $\beta = 15°...50°$, and even at $x = 0$, the ratio $V_{S\beta}/V_{S0}$ much less than 1 in comparison to the one at $\beta = 0°...15°$. In the transverse plane at $\beta = 50°$ the influence of profile shifting on the value of V_S is the highest, but it is insignificant. The influence of x coefficient on η_2 is the same, but the magnitude of $\eta_{2\beta}/\eta_{20}$ ratio is different (Fig. 10g). The ratio $V_{1\beta}/V_{10}$ is very close to 1 at $\beta = 5°$ in the range of $x_1 = -0.6...+0.8$ (Fig 10b). With the increasing of the value of β angle, the ratio $V_{1\beta}/V_{10}$ is more different from 1 at negative shift and closer to 1 at positive shift on the pinion. In the transverse plane at $\beta = 50°$ the value of $V_{1(50)}/V_{1(0)}$ is much larger than $V_{1(5)}/V_{1(0)}$ at $x_1 = -0.6$ while the values of $V_{1(50)}/V_{1(0)}$ and $V_{1(5)}/V_{1(0)}$ are close at $x_1 = +0.8$. The difference like this can be explained by the fact that the gear pair with $z_1 = 28$ is close to undercutting at $\beta = 0°$ and $x_1 = -0.6$. Unlike $V_{1\beta}/V_{10}$, the ratio $V_{2\beta}/V_{20}$ is more different from 1 at positive shift and closer to 1 at negative shift on the pinion (Fig. 10c) with the increase of the value of β angle in the range of $\beta = 15°...50°$. At small values of β angle ($\beta = 0°...15°$), the ratio $V_{2\beta}/V_{20}$ is almost constant and very close to 1 in the range of $x_1 = -0.6...+0.8$ (Fig. 10c). The nature of curves of V_Σ ratios is different: all the ratios $V_{\Sigma\beta}/V_{\Sigma 0}$ are almost independent on x coefficient (Fig. 10d). Fig. 10e shows that χ_β/χ_0 ratio changes with the nature of curves which is similar to the one of $V_{1\beta}/V_{10}$. The difference is that $V_{1\beta}/V_{10} > 1$ and the ratio $V_{1\beta}/V_{10}$ reaches its maximum level at $x_1 = -0.6$, while $\chi_\beta/\chi_0 < 1$ and the ratio χ_β/χ_0 reaches its minimum level at the same shift. The ratio χ_β/χ_0 is almost constant and close to 1 in the range of $x_1 = -0.6...+0.8$ at $\beta = 5°$ (Fig. 10e). With the increasing of the value of β angle the ratio χ_β/χ_0 is more different from 1 at negative shift and closer to 1 at positive shift on the pinion. At $x_1 = +0.8$ the ratio χ_β/χ_0 almost reaches 1 independently on the value of β angle (Fig. 10e). Unlike χ_β/χ_0, the ratio $\eta_{1\beta}/\eta_{10}$ does not reach 1 at $\beta = 15°...50°$. The other results on χ_β/χ_0 are also common for $\eta_{1\beta}/\eta_{10}$ (Fig. 10f). The ratio of ν_β/ν_0 is predictably close to 1 at $\beta = 5°$ and $x_1 = +0.8$ (Fig. 10f), moreover, the ratio's curve is slightly inclined about the axis of abscissas. The angle of curve's inclination increases along with the increase of the value of β angle (Fig. 10f).

The ratio's values from Figs. 9-10 that differ significantly from 1 and correspond to a different combination of parameters x and β are given in Tables 1 and 2.

Table 1 – Extreme values of characteristics' ratios at the extreme values of x coefficient

x_1	V_{Sx}/V_{S0}	V_{1x}/V_{10}	V_{2x}/V_{20}	$V_{\Sigma x}/V_{\Sigma 0}$	χ_x/χ_0	η_{1x}/η_{10}	η_{2x}/η_{20}	ν_x/ν_0
−0.6	1.52÷1.56	0.39÷0.76	1.07÷1.12	0.93÷0.95	1.21÷2.28	2.08÷3.87	1.35÷1.47	0.97÷1
+0.8	0.21÷0.22	1.34÷1.92	0.82÷0.9	1.07÷1.11	0.64÷0.85	0.11÷0.15	0.23÷0.27	1÷1.04

Table 2 – Extreme values of characteristics' ratios at the extreme values of β angle

β	$V_{S\beta}/V_{S0}$	$V_{1\beta}/V_{10}$	$V_{2\beta}/V_{20}$	$V_{\Sigma\beta}/V_{\Sigma 0}$	χ_β/χ_0	$\eta_{1\beta}/\eta_{10}$	$\eta_{2\beta}/\eta_{20}$	ν_β/ν_0
5°	1	1	1	1	1	1	1	1
50°	0,71÷0,76	1,53÷4,23	1.21÷1.39	1.45÷1.54	0.39÷0.97	0.18÷0.46	0.51÷0.65	0.92÷1

CONCLUSION

A group of geometric-kinematic meshing characteristics is calculated and analyzed for profile shifted quasi-involute ATT gears. The conclusions are as follows:

1. The profile shifting compared to the increasing of the value of the tooth trace angle generally effects more the values of sliding velocity and specific sliding. The improvement of rolling velocities of mating teeth and relative curvature in contrast to the sliding characteristics is more effective by increasing of the value of the tooth trace angle. The angle between sliding velocity vector and contact line is almost insensitive to both of profile shifting and increasing of β angle.

2. The detailed quantitative assay of meshing characteristics shows that the value of profile shift coefficient at the range of the tooth trace angle of $\beta = 0...15°$ has more impact on the meshing characteristics than at $\beta = 15...50°$. This feature of the meshing characteristics distribution opens the way to separate further research into two directions: (i) research of profile shifted quasi-involute ATT gears with small gear width as an alternative to the narrow spur gears and (ii) research of inclined-arc-tooth-trace gears without profile shifting as an alternative to double helical gears.

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