On endomorphisms of free commutative monogenic trioids

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An algebraic system $(T, \dashv, \vdash, \perp)$ with three binary associative operations $\dashv$, $\vdash$, and $\perp$ is called *a trioid* [1] if for all $x, y, z \in T$,

$$\begin{aligned} (x \dashv y) \dashv z &= x \dashv (y \vdash z), & (x \vdash y) \dashv z &= x \vdash (y \dashv z), \\ (x \dashv y) \vdash z &= x \vdash (y \vdash z), & (x \dashv y) \dashv z &= x \dashv (y \perp z), \\ (x \perp y) \dashv z &= x \perp (y \dashv z), & (x \dashv y) \perp z &= x \perp (y \vdash z), \\ (x \vdash y) \perp z &= x \vdash (y \perp z), & (x \perp y) \vdash z &= x \vdash (y \vdash z). \end{aligned}$$

We observe that trioids are a generalization of dimonoids and semigroups. A trioid $(T, \dashv, \vdash, \perp)$ is called *commutative* [2] if $x * y = y * x$ for any operation $* \in \{\dashv, \vdash, \perp\}$.

First, we present a trioid construction (more convenient) isomorphic to the free commutative monogenic trioid from [2]. Further, we define all endomorphisms of the free commutative monogenic trioid and describe a semigroup which is isomorphic to the endomorphism semigroup of the free commutative monogenic trioid. Note that the endomorphism monoid of a free trioid of rank 1 was described in [3].

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2. Zhuchok A. V. Free commutative trioids. Semigroup Forum, 2019, **98** (2), 355–368.
3. Zhuchok Yu. V. The endomorphism monoid of a free trioid of rank 1. Algebra Univers., 2016, **76** (3), 355–366.