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On free abelian trioids

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Trioids were introduced in [1] at the study of ternary planar trees. An algebraic system $(T, \dashv, \vdash, \perp)$ with three binary associative operations $\dashv$, $\vdash$ and $\perp$ is called *a trioid* if for all $x, y, z \in T$,

$$\begin{gathered}
(x \dashv y) \dashv z = x \dashv (y \vdash z), \quad (x \vdash y) \dashv z = x \vdash (y \dashv z), \\
(x \dashv y) \vdash z = x \vdash (y \vdash z), \quad (x \dashv y) \dashv z = x \dashv (y \perp z), \\
(x \perp y) \dashv z = x \perp (y \dashv z), \quad (x \dashv y) \perp z = x \perp (y \vdash z), \\
(x \vdash y) \perp z = x \vdash (y \perp z), \quad (x \perp y) \vdash z = x \vdash (y \vdash z).
\end{gathered}$$

Trioids are a basis of the notion of a trialgebra [1], besides they are a generalization of dimonoids [2] and semigroups. A trioid $(T, \dashv, \vdash, \perp)$ we call *abelian* if $x \dashv y = y \vdash x$ for all $x, y \in T$. Abelian trioids with a commutative operation $\perp$ were considered in [3]. A trioid which is free in the variety of abelian trioids will be called a *free abelian trioid*.

We denote by $F(X)$ and $FCm(X)$ the free semigroup on X and, respectively, the free commutative monoid on X. Further, we put

$$FAt(X) = F(X) \times FCm(X)$$

and define three binary operations $\dashv$, $\vdash$ and $\perp$ on $FAt(X)$ by

$$\begin{aligned}
(u, v) \dashv (p, q) &= (u, vpq), \\
(u, v) \vdash (p, q) &= (p, quv), \\
(u, v) \perp (p, q) &= (up, vq).
\end{aligned}$$

Theorem 1. *The algebra $(FAt(X), \dashv, \vdash, \perp)$ is the free abelian trioid.*

An addition, we describe the least congruence on a free trioid such that the corresponding quotient-trioid and the free abelian trioid are isomorphic. Note that the construction of the free abelian dimonoid and some its properties were presented in [4].

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[2] Loday J.-L., Dialgebras, in: Dialgebras and related operads, *Lect. Notes Math.* **1763**, Springer-Verlag, Berlin, 2001, 7–66.
[3] Huang J., Chen Yu., Gröbner-Shirshov bases theory for trialgebras, *Mathematics* **9** (2021), 1207. https://doi.org/10.3390/math9111207.
[4] Zhuchok Yu.V., Free abelian dimonoids, *Algebra Discrete Math.* **20** (2015), no. 2, 330–342.

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