Emil Artin International Conference

Dedicated to the 120th Anniversary of Emil Artin
(03.07.1898-20.12.1962)

Yerevan, the Republic of Armenia,
May 27-June 2, 2018.

ABSTRACTS

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Yerevan 2018

EMS Conference

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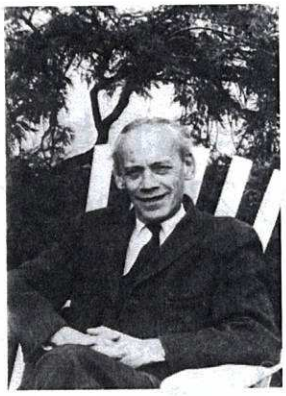

The conference is organized by:

- Armenian Mathematical Union
- Yerevan State University
- American University of Armenia
- Committee of Emil Artin Junior Prize in Mathematics
- Institute of Mathematics of National Academy of Sciences of Armenia
- University of Bergen
- Steklov Mathematical Institute of the Russian Academy of Sciences

Conference Topics:

- Classical and non-classical Algebraic Structures,
- Algebra and Logics Number Theory,
- Geometry and Topology, Analysis and Equations,
- Boolean and De Morgan functions Cryptography and Discrete Mathematics, Applied Mathematics,
- Lattices, Universal Algebra, Computer Science and IT,
- Artin L functions, Dynamical Systems
- Quantum information theory, Quantum Logic and Quantum Computation, Quantum Groups and Quantum Quasigroups.

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CONTENTS

ON FREE k-NILPOTENT n-TUPLE SEMIGROUPS

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Following [1], a nonempty set G equipped with n binary operations $\boxed{1}, \boxed{2}, ..., \boxed{n}$, satisfying the axioms $(x\boxed{r}y)\boxed{s}z = x\boxed{r}(y\boxed{s}z)$ for all $x, y, z \in G$ and $r, s \in \{1, 2, ..., n\}$, is called an n-tuple semigroup. An element 0 of an n-tuple semigroup $(G, \boxed{1}, \boxed{2}, ..., \boxed{n})$ will be called zero if $x * 0 = 0 = 0 * x$ for all $x \in G$ and $* \in \{\boxed{1}, \boxed{2}, ..., \boxed{n}\}$. An n-tuple semigroup $(G, \boxed{1}, \boxed{2}, ..., \boxed{n})$ with zero 0 will be called nilpotent if for some $m \in \mathbb{N}$ and any $x_i \in G$ with $1 \leq i \leq m+1$, and $*_j \in \{\boxed{1}, \boxed{2}, ..., \boxed{n}\}$ with $1 \leq j \leq m$,

$$x_1 *_1 x_2 *_2 \ldots *_m x_{m+1} = 0.$$

The least such m will be called the nilpotency index of $(G, \boxed{1}, \boxed{2}, ..., \boxed{n})$. For $k \in \mathbb{N}$ a nilpotent n-tuple semigroup of nilpotency index $\leq k$ is said to be k-nilpotent.

An n-tuple semigroup which is free in the variety of k-nilpotent n-tuple semigroups will be called a free k-nilpotent n-tuple semigroup. If ρ is a congruence on an n-tuple semigroup G' such that G'/ρ is a k-nilpotent n-tuple semigroup, we say that ρ is a k-nilpotent congruence.

Let X be an arbitrary nonempty set and ω an arbitrary word in the alphabet X. The length of ω will be denoted by l_ω. Fix $n \in \mathbb{N}$ and let $Y = \{y_1, y_2, ..., y_n\}$ be an arbitrary set consisting of n elements. Let further $F[X]$ be the free semigroup on X, $F^\theta[Y]$ the free monoid on Y and $\theta \in F^\theta[Y]$ the empty word. Fix $k \in \mathbb{N}$ and define n binary operations $\boxed{1}, \boxed{2}, ..., \boxed{n}$ on

$$XY_{[k]} = \{(w, u) \in F[X] \times F^\theta[Y] \mid l_w - l_u = 1,\ l_w \leq k\} \cup \{0\} \quad \text{by}$$

$$(w_1, u_1)\boxed{i}(w_2, u_2) = \begin{cases} (w_1w_2, u_1y_iu_2), & l_{w_1w_2} \leq k, \\ 0, & l_{w_1w_2} > k, \end{cases}$$

$$(w_1, u_1)\boxed{i}0 = 0\boxed{i}(w_1, u_1) = 0\boxed{i}0 = 0$$

for all $(w_1, u_1), (w_2, u_2) \in XY_{[k]} \backslash \{0\}$ and $i \in \{1, 2, ..., n\}$. The algebra obtained in this way will be denoted by $FN_n^kS(X)$.

Theorem. *$FN_n^kS(X)$ is the free k-nilpotent n-tuple semigroup.*

Corollary. *The free k-nilpotent n-tuple semigroup $FN_n^kS(X)$ generated by a finite set $X \times \{\theta\}$ is finite. Specifically, $|FN_n^kS(X)| = \sum_{i=1}^{k} n^{i-1}|X|^i + 1$.*

We also consider separately one-generated free k-nilpotent n-tuple semigroups and describe the least k-nilpotent congruence on a free n-tuple semigroup [2].

References

[1] Koreshkov N.A., *n-Tuple algebras of associative type*, Russian Mathematics **52** (12), 2008, 28–35.

[2] Zhuchok A.V., *Free n-tuple semigroups*, Math. Notes **103** (5), 2018, 693–701 (in Russian).