#### МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

# ПОЛТАВСЬКИЙ НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ імені Юрія Кондратюка

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КАФЕДРА КОНСТРУКЦІЙ З МЕТАЛУ, ДЕРЕВА ТА ПЛАСТМАС

#### Збірник текстів

до теми

# Reliability of Building Structures

з дисципліни "Англійська мова"

для студентів, дипломників і магістрантів всіх спеціальностей будівельного факультету, аспірантів і здобувачів наукового ступеня кандидата технічних наук

Збірник текстів до теми "Reliability of Building Structures" з дисципліни "Англійська мова" для студентів, дипломників і магістрантів всіх спеціальностей будівельного факультету, аспірантів і здобувачів наукового ступеня кандидата технічних наук / Полтава, Полтавський національний технічний університет імені Юрія Кондратюка, 2005р. — 000с.

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#### ВСТУП

Дане методичне видання містить результати багаторічної сумісної роботи кафедр іноземної мови та конструкцій з металу, дерева та пластмає університету і включає підготовлені викладачами цих кафедр матеріали і наукові доповіді, представлені на міжнародних конференціях, що проводилися у зарубіжних країнах, таких як Італія, Норвегія, Швейцарія, Мальта, Чехія, Словакія, Угорщина, Польща. Вміщені у даному збірнику тексти доповідей опубліковані у збірниках праць відповідних конференцій.

Автори вважають, що наведені у даному виданні тексти мають подвійне наукове-методичне призначення.

По-перше, вони дають достатньо повне уявлення про зміст науковопрактичного методу оцінки надійності будівельних конструкцій, розробленого в одній із наукових шкіл ПНТУ імені Юрія Кондратюка. У цьому відношенні наведені матеріали можуть бути корисними, зокрема, науковцям-початківцям у входженні в актуальну проблематику надійності у будівництві та у виборі напряму подальших наукових досліджень. Суть і особливості розробленого методу розкриваються у логічній послідовності у наступних розділах:

- загальний метод розрахунку надійності будівельних конструкцій (розділ І);
- імовірнісний опис навантажень на будівлі і споруди вітрового (розділ ІІ), снігового (розділ ІІІ) і кранового (розділ IV) навантажень;
- оцінка надійності сталевих елементів і статично невизначених систем (розділ V).

По-друге, з методичної і лінгвістичної точки зору представлені тексти можуть використовуватися :

- для вдосконалення навичок читання науково-технічної літератури та роботи з різними словниками і довідковою літературою;
- у розвитку навичок письма з опорою на прочитаний текст;
- для розширення активного і пасивного термінологічного словника читача;
- з метою вдосконалення навичок технічного перекладу;
- як показові приклади наукових публікацій по технічній тематиці, які можна застосовувати, зокрема, у підготовці доповідей на міжнародних конференціях;
- як зразки стилю викладення, представлення і оформлення наукових публікацій за кордоном;
- як тексти самостійної підготовки студентів і аспірантів, зважаючи на дефіцит оригінальної наукової літератури англійською мовою.

Варто відмітити той факт, що великий об'єм англомовного матеріалу, зібраний в даному збірнику, дозволяє формувати і розвивати навички реферування і анотування, без володіння якими успішне навчання і наукова діяльність стають проблематичними.

Даний збірник текстів у повній мірі адресується студентам, дипломникам і магістрантам всіх спеціальностей будівельного факультету, аспірантам і здобувачам наукового ступеня кандидата технічних наук будівельного напряму.

Крім того, зважаючи на широке використання математичних й імовірнісних методів, представлені матеріали можуть використовуватися також студентами і аспірантами напрямів "Прикладна математика" та "Комп'ютерна техніка".

Автори вважають, що запропонований збірник текстів буде сприяти формуванню позитивної мотивації учення, так як успішне оволодіння складними навичками розуміння іншомовного тексту допоможе тим, хто навчається, піднятися ще на одну сходинку вище по крутим сходам наукового пізнання і професійного удосконалення.

Даний збірник текстів може бути використаний як для навчання читанню в аудиторії під керівництвом викладача, так і для самостійної роботи студентів старших курсів, аспірантів, пошукувачів.

Автори

# GENERAL METHOD OF STRUCTURE RELIABILITY ESTIMATION

# RELIABILITY ESTIMATION OF STEEL ELEMENTS UNDER VARIABLE LOADS

#### 1 INTRODUCTION

The probabilistic assessment of structural elements with random strengths subject to snow, wind and crane loads is under investigation. The actions are described by random processes (RP). Their real distributions and frequency characteristics are taken into account.

#### 2 GENERAL APPROACH

The principal ideas of applied method were developed in works [1, 2]. The failure of an element takes place when a stochastic stress under the joint applied loads S(t) (t-time) exceeds the random limit of a steel yielding state (resistance of an element) R. The failure of the element is defined by the equation

$$\widetilde{Y}(t) = \widetilde{R} - \widetilde{S}(t) < 0, \tag{1}$$

where: Y(t) — margin function of load carrying capacity introduced by A.R.Rzhanitsyn [3].

The safety characteristic is of great importance and is derived from equation

$$\beta = \overline{Y} / \hat{Y} = (\overline{R} - \overline{S}) / (\hat{R}^2 + \hat{S}^2)^{1/2}, \tag{2}$$

where:  $\overline{Y}, \overline{R}, \overline{S}$  - the corresponding mathematical expectations,  $\hat{Y}, \hat{R}, \hat{S}$  — the corresponding standard mean deviations.

The loads are presented in the form of stationary and quasistationary RP. Hence the Y(t) function is RP as well. Therefore the element failure is a projection of RP load carrying capacity margin (1) into the negative region. In this case the probability of failure Q(t) is estimated as follows

$$Q(t) = \varpi_{a} \cdot f(\beta)t/(\beta_{\pi}\sqrt{2\pi}), \tag{3}$$

where:  $\omega_q$  and  $f_Y(z)$  — the effective frequency and density of the distribution of RP Y(t) ordinate;  $\beta_m$  — the coefficient of RP  $\tilde{Y}(t)$  structural complexity.

The equation (3) is based on the formula of frequency of stationary RP projection which has the general distribution of ordinate and normal distribution of a derivative [4].

The solving of the problem (3) demands the presentation of distributions and the characteristics of RP  $\tilde{Y}(t)$  frequency.

#### 3 JOINT LOAD AND STRENGTH DISTRIBUTIONS DESIGN

Various statistical distributions were used in probabilistic research [5]. The normal distribution was introduced for a steel yielding state and crane loads. The Weibull's distribution was applied [6] for the wind. The snow load for changeable

Ukrainian winters was described by exponential law with the argument in the form of a polynomial of the 3-th degree (fig. 1b).

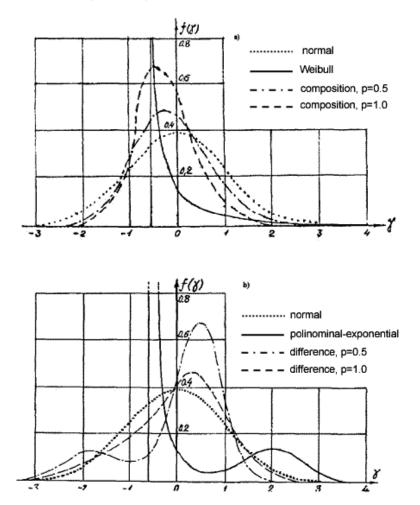


Fig. 1. Examples of joint load and strength distributions: a) composition of the normal and Weibull's distributions, b) the normal and polynomial — exponential distribution.

The compositions of differences of loads and strength should be done several loads and for design of margin of carrying capacity (1). Convolution formulae are used for this purpose. They can not be determined in a closed form for the mentioned distributions therefore numerical integration is adopted. The convolutions are transformed (presented in an unified comparable form) for the design simplification and their parameters are combined. From 13 received analytical expressions of convolutions, only the formula for difference of normal and arbitrary  $f_2(z)$  density distributions is suggested

$$f(\gamma) = (D/\sqrt{2\pi}) \int_{z_1}^{z_2} \exp(-0.5E^2) f_2(z) dz,$$
 (4)

where:  $y = (Y - \overline{Y})/\hat{Y}$  — standardised variable;  $E = D \cdot \gamma + z/p$ ;  $D = \sqrt{1 + p^2}$ ;  $p = \hat{x}_n/\hat{x}_a$ ;  $v_n = \hat{x}_n/\bar{x}_n$ ;  $\bar{x}_n$ ;  $\bar{x}_n$  — parameters of the normal distribution,  $\hat{x}_a$  — standard

deviation of arbitrary distribution,  $z_1 = \gamma \sqrt{1 + p^2} - p/v_n$  — lower limit of integration,  $z_2$  — upper limit of integration which is determined with the account of necessary calculation accuracy.

The examples of joint distributions based on the proposed formulae are presented on the figure 1. They can be non-symmetrical and differ considerably from the normal distribution in solutions of structural reliability problems.

## 4 ANALYSIS OF FREQUENCY COMPOSITION OF SEVERAL SIMULTANEOUS APPLIED LOADS

The overhead-crane load is a stationary RP. Snow and wind loads are the quasistationary RP's with a constant effective frequency and mathematical expectation which changes slowly during the year. This change is supposed to be accounted with  $k_u$ . coefficients which are in the limits of 0.35-0.42 for wind and for snow it is 0.13-0.23 (for Ukraine). With regard to the above mentioned information the known formula of effective frequency summation [4] can be written in a following way:

$$\omega_q = \left[ \sum_{i=1}^n (\alpha_i \cdot \hat{q}_i \cdot \omega_i \cdot k_{tri})^2 / \sum_{i=1}^n (\alpha_i \cdot \hat{q}_i)^2 \right]^{1/2}, \tag{5}$$

where:  $\omega_q$ ,  $\omega_i$  — effective joint and *i*-th load frequencies, respectively,  $\alpha_t$ , q, — influence number and i-th load standard.

Correlation functions (fig. 2a) and effective frequencies of RP various loads differ considerably: for crane loads  $\omega = 1700 \div 5160$  1/day, for wind ones  $\omega = 5.4 \div 6.6$  1/day, for snow loads  $\omega = 0.073 \div 0.141$  1/day [2]. Therefore the joint action of several loads become multifrequent. This peculiarity can be defined in the formula (3) by the RP structure complexity coefficient which equals the ratio of mean frequency on maximum  $\omega_m$  to the effective frequency on zero  $\omega_q$ :

$$\beta = \frac{\omega_m}{\omega_q} = \frac{\left[K^{IV}(0) \cdot K(0)\right]^{1/2}}{\left[-\ddot{K}(0)\right]} = \frac{\beta_i \cdot \left[(1 + k^2 \cdot \Theta^4)(1 + k^2)\right]^{1/2}}{(1 + k^2 \cdot \Theta^2)},\tag{6}$$

where: K(0),  $\ddot{K}(0)$ ,  $K^{IV}(0)$  — RP correlation function and its derivatives with zero argument,  $\beta_i$  — RP structure complexity coefficient of separate loads equals approximately 3,  $k = \hat{x}_2/\hat{x}_1$ ;  $\theta = \omega_2/\omega_1$  — are the standard deviations and effective frequency proportion of summed up RP loads.

In accordance with equations (6) the frequency composition analysis can be realised if RP is differentiated sufficiently many times.

For the atmospheric and crane loads the coefficient of structure complexity is shown at figure 2b, which is done on a logarithmic scale. The  $\beta_{\omega}$  — coefficient differs negligibly from  $\beta_t$ =3 coefficient which is usual for separate RP with  $k \ge 1$ , it goes up synchronously at the interval k= $l.0 \div 0.1$ ; it gets maximum in a narrow band k= $l.0.2 \div l0.5$  and it goes down again to  $\beta_t$ =3 with k=0. If the effective frequency of one of the loads prevails and  $\omega$  is large, then formula (6) is simplified as follows:

$$\beta_{\omega} = \beta_i (1 + k^2)^{1/2} / k,$$
 (7)

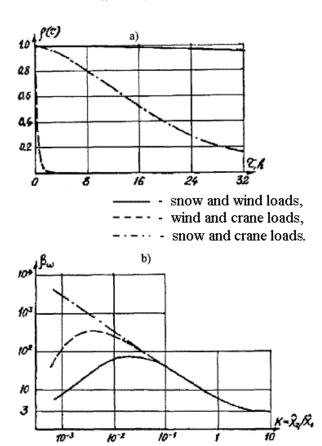


Fig. 2. The frequent analysis of load combination: a) normalised correlation functions, b) structural complexity coefficient of combined loads

#### 5 CONCLUSION

The worked out method gives possibility to estimate reliability of a wide range of steel structures and to specify some coefficients of design standard [7].

#### 6 REFERENCES

- [1] Pichugin S.: Reliability Design Method of Steel Structures. XL Scientific Conference of Civil Engineering Committee Polish Academy of Science and Scientific Committee of PZITB. Rzeszow-Krynica-Warszawa, 1994 (in Russian).
- [2] Pichugin S.: Industrial Steel Structures Reliability. Synopsis of Thesis for Doctorate of Technical Sciencies, Kiev, 1994 (in Russian).
- [3] Rzhanitsyn A.: The Design Theory of Building Structures Reliability. Moscow, Stroyiz-dat, 1978 (in Russian).
- [4] Bolotin V.: Statistical Methods in Structural Mechanics. Holden-Day, San Francisco, 1969.
- [5] Pichugin S.: The Probabilistic Load Demonstration Applied to Building Structures. I-zvestiya Vuzov (The News from High Educational Establishments) Construction, Num.4, 1995 (in Russian).
- [6] Pashinski V.A., Pichugin S.F.: Wind Load Probabilistic Description and Value Computation Procedure Adopted for Building Code of Ukraine. East European

- Conference on Wind Engineering "EECWE'94", Preprints, Part 1, Vol.3, Warsaw, 1994.
- [7] Pichugin S.: Calculated Coefficients of Designed Norms Based on the Steel Structure Reliability Analysis. Construction in Ukraine, Num.1, 1994 (in Ukrainian).

# GENERAL METHOD OF STRUCTURE RELIABILITY ESTIMATION

#### **SUMMARY**

The probabilistic method of structural reliability estimation has been developed. It takes account of the random load and steel strength, loads joint action, specific character of work and failure of steel elements, nodes and the whole steel structure as well. On the base of the determined method the numerical computations of a wide range of steel structures including crane, girders, trusses, beams, columns, frames were realized. As a result some design coefficients for Codes improvement as well as the reduction of cost have been offered.

Keywords: reliability, failure, load, structure, crane girders, truss, beam, column, frame.

#### 1 INTRODUCTION

The steel structure reliability is the problem of high priority today. It can be treated like a scientific trend, which doesn't need both material and financial expenses. The general probabilistic models of wind, snow and crane loads can be considered as a statistic basis of reliability estimation. However, these approaches lack both systematic analysis and model comparison, which can cause different results in structure reliability design. The numeric values of wind and snow loads probabilistic parameters were not determined for different districts of Ukraine. The probabilistic computation of structures was not developed with regard to join loads application, their real distributions and frequency characteristics. The assessment of a redundant structure safety is rather difficult task. In spite of many studies that have been done the problem of reliability analysis of these structures has not been solved yet. The exploitation experience demonstrated the existence of quite different reliability of various structures.

#### 2 BASE OF PROPOSED METHOD

#### 2.1 Researches of Loads

The general algorithm of the method of reliability estimation of steel building structures is illustrated in figure 1. This method is based on experimental researches, pooling and integration of load statistic data. The results of regular snow measurements for 15–40 years at 62 Ukrainian meteorological stations have been taken as reference statistical material for ground snow load. The systematic information about the wind velocity measurements done with ten minutes average at 70 Ukrainian and

NIS (New Independent States) meteorological stations were used as a initial data. The wind force values were analyzed with the help of a certain quadratic transformation of the wind velocity without the account of its direction. The crane load experimental studies were performed at several metallurgical plants in different shops from 10 to 30 years of service. Vertical and horizontal loads of bridge cranes with rigid or flexible hanged cargo were analyzed.

In accordance with obtained results the following load features were determined. Ground snow and wind loads for Ukraine are of a quasi-stationary origin. Their mathematical expectations and standards have a seasonal trend. At the same time snow and wind frequent characteristics and normalized ordinate distributions remain constant during the season [1,2]. The crane load is stationary and ergodic; its density distribution corresponds well to normal law [3]. Taking into account the bimodal characters character of Ukrainian snow density distributions so-called polinomial-exponential law was used:

$$f(\gamma) = \exp(C_0 + C_1 \gamma + C_2 \gamma^2 + C_3 \gamma^3), \tag{1}$$

where  $\gamma = (x - \overline{X})/\hat{X}$  — normalized deviation of snow load;  $\tilde{N}_0, ..., C_3$  — parameters which are determined by the values of load mathematical expectations, standard and relative skewness. Wind density distribution is well approximated by Veibull's law or distributions (1).

#### 2.2 Probabilistic Load Models

The systematic analysis of random loads was realized for the five most commonly used probabilistic models. The main one is presented in the form of a stationary (crane load) or quasi-stationary (wind and snow loads) random processes; their parameters are an effective frequency  $\omega$  and the coefficient of trend  $K_{tr}$  which accounts the atmospheric load season change.

The absolute maxima of random process are one of simpler model; they are determined by the tail part of the distribution of outliers and are higher then characteristical maximum level  $\gamma_0$ . The letter is a solution of the following equation  $N_+(\gamma_0; 0 \le \tau \le t) = 1$ , where  $N_+(\cdot)$  — the number of outliers of random process.

The model in the form of a random sequence of independent random loads with the intensity  $\lambda$  is widely spread ( $\lambda$  — the number of loads in per-unit time  $t_{\lambda}$ ). For discrete presentation of loads the frequent parameter of which is the mean duration of overloading  $\overline{\Delta}$  connected with the intensity by the ratio  $\overline{\Delta} = t_{\lambda} / \lambda$ .

The analysis of the problem has demonstrated that the load values sampling can be classified as the exponential type. That's why their maximum values can be presented correctly by the extreme double exponential Gumbel distributions of a normalized type:

$$y = \alpha_n(\gamma - u_n), \qquad (2)$$

where  $u_n$  — characteristic extremum;  $\alpha_n$  — extreme intensity;  $y = -\ln[-\ln F(t)]$  — Gumbel distribution argument; F(t) — integral distribution function.

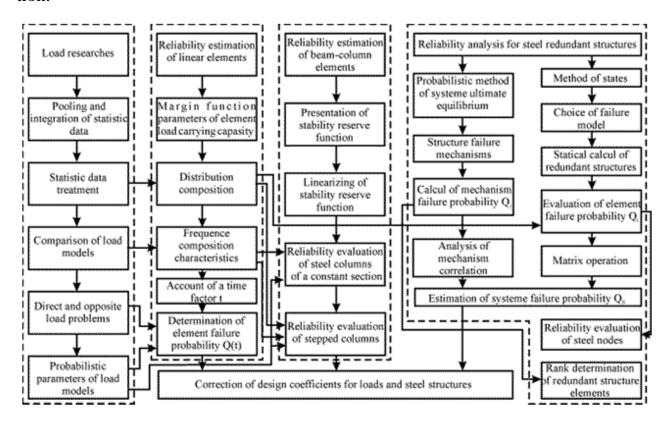


Fig. 1. General algorithm of reliability estimation of steel building structures.

#### 2.3 Comparison of Load Models

The choice of models of loads depends on a specific of solving probabilistic problems: the more complicated ones are solved in the manner of random processes which, however, are difficult for description and take much more computation time. More accessible and simple models, mentioned earlier, are based on the random values and corresponding frequent characteristics and provide not less exact solution if they are proper grounded. In particular, all the examined models are close to its sense of the following evaluation:

$$Q(t) \cong N_{+}(\gamma, t) = \omega t K_{tr} f(\gamma) / \sqrt{2\pi} = f(\gamma) / f(\gamma_0) =$$

$$= \lambda t [1 - F(t)] = t [1 - F(\gamma)] / \overline{\Delta},$$
(3)

where Q(t) — probability of exceeding  $\gamma$  — level during t.

The first formula is based on the expression of frequency of quasi-stationary random process which has the general distribution of ordinate and normal distribution of a derivative [4].

Provided with the equation (3) the formulae which connect the parameters of different probabilistic load models were derived in author's papers [1,2,5].

Load probabilistic comparison can be well performed at the extreme scale [6] which is illustrated in figure 2. On the axis of ordinate of a scale the standardized

load is laid off, on the axis of abscissa the Gumbel's distribution argument y is laid of which is connected with the load return period T.Gumbel's distribution (2) is described on the scale in the form of straight lines. The models of a random process, a random sequence of independent loads and discrete presentation are introduced as different curves. The main advantage of this scale is its visual effect of the tail parts of the load distributions which have rather small distinctions in the usual form of presentation. It enables to present the visual comparison and correspondence of parameters of different load models.

All necessary mean wind and snow probabilistic parameters of Ukrainian districts were introduced in our works [1,2]. The worked out parameters give possibility to develop the reliability estimation of building structures.

### 3 RELIABILITY OF STEEL ELEMENTS UNDER DIFFERENT RANDOM LOADS

#### 3.1 General Approach

The principal ideas of applied method were developed in works [7,8]. The failure of an element takes place when a stochastic stress under the joint applied loads  $\tilde{S}(t)$  exceeds the random limit of a steel yielding state (resistance of an element  $\tilde{R}$ ). The failure of the element is defined by the equation

$$\widetilde{Y}(t) = \widetilde{R} - \widetilde{S}(t) < 0$$
, (4)

where  $\tilde{Y}(t)$  — margin function of load carrying capacity.

The safety characteristics is of great importance and is derived from equation

$$\beta = \overline{Y} / \hat{Y} = (\overline{R} - \overline{S}) / (\hat{R}^2 + \hat{S}^2)^{1/2},$$
 (5)

where  $\overline{Y}, \overline{R}, \overline{S}$  — the corresponding mathematical expectations;  $\hat{Y}, \hat{R}, \hat{S}$  — the corresponding standard mean deviations.

The probability of failure Q(t) is estimated from the formula (3). The solving of this problem demands the presentation of joint distributions and the characteristics of  $\tilde{Y}(t)$  frequency. The compositions or differences of load and strength distributions should be done for design of margin of carrying capacity (5). Convolution formulae are used for this purpose; their analytical expressions were represented in our paper [8]. The examples of joint distributions based on the proposed formulae are presented on the figure 3. They can be non-symmetrical and differ considerably from the normal distribution in solutions of structural reliability problems.

Correlation functions and effective frequencies of random processes of various loads differ considerably [7,8]. Therefore the joint action of several loads becomes multi-frequent. This peculiarity can be defined in the formula (3) by the structure complexity coefficient  $\beta_{\omega}$  which equals the ratio of mean frequency on maximum to the effective frequency on zero.

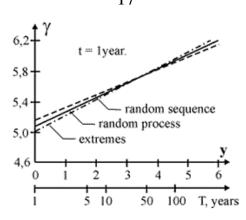


Fig. 2. Comparison of crane load models

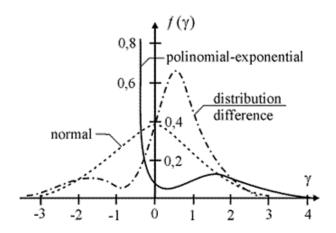


Fig. 3. Difference of distributions.

#### 3.2 Analysis of Steel Element Reliability

Some of the building parts consist of numerous structures, roofs or ceilings. The dead load of structures like these is a sum of normal random values describing the weight of every layer. It is established that decreasing coefficient of combination  $\psi$ =0.60-0.95 can be applied to a dead load in this case.

Crane beams as well as crane trestle structure can be treated like elements under only crane load. We'll examine general stress of these elements without local effects and fatigue. In this case the probability of structure failure Q(t) depends on the ratio of a load stress of one bridge crane  $X_{M1}$  to the load stress of two bridge cranes  $X_{M2}$ , then on the ratio of a load carrying capacity of cranes to their mass and it also depends on crane work condition. The analysis showed that steel elements have a deficient reliability if they are under 4-wheeled crane load when  $X_{M1}/X_{M2} \ge 0.8$  i.e. in the case of the one crane loads dominance. In the rest cases steel elements reliability determined on the base of general stress state criteria under crane loads is sufficient. In some cases the design crane load on steel structures can be decreased when their reliability is preserved. For example in the case if the approach of a loaded crane trolley to a crane track is limited or if the structure service term T is also limited. For the last case the design crane load may be multiplied by temporary coefficient  $\gamma_T = 0.93-1.0$  for T = 1-50 years.

The reliability of steel structures (beams, trusses) under snow load was estimated by a developed method. These structures are of different mass roofs and snow

loads for all Ukrainian districts. The calculation demonstrated the lack of reliability of rafter structures. That justifies the idea of understating of existing snow loads Code in Ukraine. Besides this fact validates the causes of steel truss failures. It applies to steel structures with lightweight roofs in the southern districts of Ukraine when there is much snow in winter. The increasing of snow design load for 1.5-2 times for our region can solve the reliability problem of steel structures under snow load.

Steel elements under wind load designed in accordance with the existing code (glass elements, wind protection screens etc) are of sufficient reliability. So if the service term of these elements will be limited it is possible to introduce the temporary coefficient  $\gamma_T$ =0.46-1.0 for T=1-50 years for the design wind load. The obtained results allow decreasing considerably the design wind load for the conditions of structure erection. It is recommended to introduce the increasing coefficients of combination  $\psi$ =0.7-0.9 into the steel structure design under snow, wind and crane loads.

#### 4 RELIABILITY OF STEEL BEAM-COLUMN STRUCTURES

Time factor, existing loads, random steel strength were taken into account during computation process of these elements [9]. For that reason the statistic stability of steel beam column elements was investigated; the function of stability reserve has been offered; the possibility of its linearizing has been suggested; some approaches with regard to geometrical and physical non-linearity have been worked out. Existing steel columns of industrial buildings in a broad range of parameters were examined: when the truss and column conjugation is rigid or hinged; if the building span is 24-36 m, column step is 6-12 m: if the roof is with steel profile flooring or reinforced concrete panels, with bridge cranes of 30 to 125 tons and different conditions of work. It is right for all snow and wind districts of Ukraine. All the columns were designed in accordance with existing Code.

The analysis of obtained results demonstrated the failure probability of steel column of a constant section as well as the upper parts of stepped columns. The failure period of mentioned above elements is 100-250 years. So the general conclusion is as follows: the reliability of steel columns of industrial buildings is quite sufficient. Besides, the reliability of lower parts of the stepped columns appeared to be much higher then upper ones. That is the result of applying additional random crane force to the lower part of a column. Taking into account these facts and using the criteria of equal reliability of upper and lower parts it is recommended to introduce the additional coefficient of work condition  $?_C = 1.15$  in the calculation of lower parts of stepped columns. The introduction of this coefficient will allow decreasing steel column weight.

#### 5 RELIABILITY ANALYSIS OF STEEL STRUCTURE NODES

The logic and probabilistic methodology was used for analysis. It was founded out that reliability of typical steel nodes could be compared with the schemes of successive connections of correlated elements. Thus, the node reliability depends upon the number of engaged independent load carrying elements. It is obvious that if the number of elements is increased the node reliability will be decreased. The reliability of nodes with less number of elements is higher. As a result of this the nodes can be

less reliable that the structure itself (column rods, span parts of beams and so on). The reliability of existing structures has to be examined together with the reliability of elements (sections) as well as nodes.

#### 6 PROBABILISTIC ESTIMATION OF STEEL REDUNDANT STRUCTURES

Some beams and simple frames, as well as multistory and multi-span structures of industrial and residential buildings present this type of structures. The failures of these systems are various. This paper attacks the problem which concern is only steel structures with the loss of carrying load capacity [10]. Redundant structure failures occur after some member failures in the form of transmission to different workable states. These states match different designing schemes with various probabilistic parameters. Thus, the redundant structure failure estimation is a very complicated problem as depends upon the system complexity. The method of states, probabilistic method of ultimate equilibrium and logic and probabilistic method were developed for solving this problem. The main attention was paid to the redundant structures with bending dominance (beams, frames). Corresponding formulae are derived; the algorithms and computing programs are developed. The estimation of a wide range of industrial redundant structures with different degree of redundancy was obtained on the base of this approach. It gave possibility to evaluate the safety level of redundant structures in comparison with separate members and statically determined structures. This level can be taken into account introducing the additional coefficient of work condition  $\gamma_C=1.1...1.4$ . The minimum values of this coefficient correspond to the least number of appeared plastic hinges (two or three for so called "partial" mechanism of structure failure). The largest coefficient  $\gamma_C$  values have to do with failure scheme with the largest number of hinges of plasticity close to the degree of redundancy ("full" failure mechanism).

#### 7 CONCLUSION

The general probabilistic models of wind, snow and crane loads were designed and taken as a statistic basis of reliability estimation. These models are as follows: stationary and quasi-stationary random processes, its absolute maxima, and random sequence of independent loads, discrete presentation and extreme model. The practical method of reliability analysis of steel structure elements is proposed. Load real distributions and frequency characteristics are taken into account. Using the worked out method the reliability analysis of wide range of steel structures designed in accordance with existing Codes was examined. This analysis showed that the structures have rather different level of reliability. In particular the light roof structures are not reliable enough being under great influence of snow load for Ukraine. At the same time the Building Code allows over-estimation of reliability for steel columns. With regard to mentioned above it's proposed to correct some load factors, a combination factor and a factor for model uncertainties of Design Codes of steel structures and loads. In addition the numeric computations offer to introduce a design coefficient, which takes into account the specific character of work and failure of redundant structures.

#### **8 REFERENCES**

- [1] Pichugin S., "Probabilistic Analysis of Wind Load and Reliability of Structures", Proc. 2-nd European and African Conf. Wind Eng., Genova, Italy, Vol. 2, SGEditoriali, Padova, 1997, p.p. 1883 1890.
- [2] Pichugin S.F., "Probabilistic Description of Snow Load for Ukraine", Proc. 1-st Intern. Scientific Technical Conf. "Technical Meteorology of the Carpathians", Publishing "OXAPT", Lviv, 1998, p.p. 85 90 (in Russian)
- [3] Pichugin S., "Analysis of Bridge Crane Loads on Industrial Buildings", XLIV Konf. Naukowa KILIW PAN i KN PZITB "Krynica 1998", Tom VII, Politechnika Poznanska, Poznan Krynica 1998, p.p. 41 47.
- [4] Bolotin V.V., Statistic Methods in Structural Mechanic, Holden Day, San Francisco, 1969, 250 p.p.
- [5] Pichugin S.F., "Probabilistic Presentation of Load Acting on Building Structures", News for High Educational Establishments. Construction, No. 4, 1995, p.p. 12 18 (in Russian).
- [6] Gumball E.I., Statistics of Extremes, Columbia University Press, New York, 1967, 320 p.p.
- [7] Pichugin S., "Reliability Estimation of Steel Elements under Variable Loads", XLI Konf. Naukowa KILIW PAN i KN PZITB "Krynica 1995", Tom III, Politechnika Krakowska, Krakow Krynica 1995, p.p. 151 156.
- [8] Pichugin S.F., "Probabilistic Design of Steel Elements for Joint Load Action", News for High Educational Establishments. Construction, No. 5,6, 1995, p.p. 23 29 (in Russian).
- [9] Pichugin S., Kharchenko Y., "Computation Approach to Probabilistic Evaluation of Steel Beam Column Elements", Proc. 6-th Intern. Conf. "Modern Building Materials, Structures and Techniques", Vol. III, "Technika", Vilnius, 1999, p.p. 21 24 (in Russian).
- [10] Pichugin S., "Probabilistic Analysis of Redundant Steel Structures", XLII Konf. Naukowa KILIW PAN i KN PZITB "Krynica 1996", Tom III, Politechnika Krakowska, Krakow Krynica 1996, p.p.85 92.

#### WIND LOAD ON BUILDING STRUCTURES

# PROBABILISTIC ANALYSIS ON WIND LOAD AND RELIABILITY OF STRUCTURES

#### **SUMMARY**

The mean value of wind load on a building structure is under investigation in this paper. The different probabilistic models of wind load were developed. The method, which connects the components of various wind models, was obtained. All necessary wind parameters were determined for districts of Ukraine. The estimation of the design values of the wind load has been substantiated.

#### 1 INTRODUCTION

The evolution of reliability computation theory and design Codes of building structures are still of interest because of the complexity of a problem on the one hand and on the other of the ignoring the random loads including the wind ones. The numeric values of wind loads probabilistic parameters were not determined for different districts of Ukraine. The traditional approach doesn't enable to get the exact structure reliability evaluation for the practical purposes especially for some building processes as reconstruction, erection and setting up the unique structures. There were used different probabilistic models for wind load description but the most of them are based on the deficient statistic material and presented in the forms of samples.

In our previous paper [1] only some problems on wind load description were examined. In this one we'll give more detailed information on this matter. The given below results are treated as the integral part of reliability estimation method which was developed in the previous works [2, 3].

#### 2 STOCHASTIC PROPERTIES OF WIND LOADS

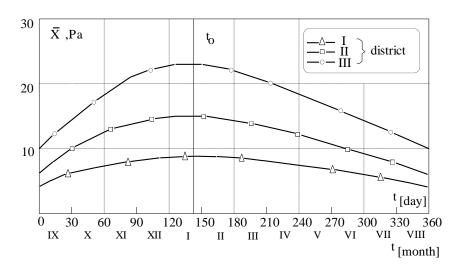


Fig. 1. Season change of the wind load mathematic expectation.

The systematic results of the wind velocity measurements are quite informative and allow getting the ordinate distribution, the numeric characteristics and frequent parameters of the mean wind value. This value corresponds to the macrometeorological peak of Vanderkhoven wind spectrum [4], with the period of 1 up 4 twenty-four hours.

Wind load has the season change of the mathematic expectation X and the standard deviation X during the year, which can be described approximately with polynomial of the 3rd degree as it's illustrated in figure 1. At the same time the coefficient of variation V, relative skewness S and excess E can be treated as time independent.

The wind load for examined territory is of the stationary frequency nature. This solution is based on the fact that its normalized correlation function and effective frequency have no significant distinctions during the year.

The experimental distributions of the wind load are well corresponded to the Veibull's law (see figure 2).

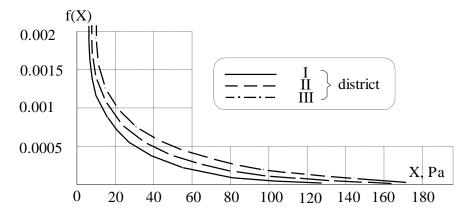


Fig. 2. Veibull's law for the wind ordinate (January).

Its integral and differential functions are usually written in the form of:

$$F(X) = 1 - \exp\left[-(X - \varepsilon)^{\beta}/\alpha\right],$$

$$f(X) = \left(\beta/\alpha\right)\left(X - \varepsilon\right)^{\beta - 1} \exp\left[-(X - \varepsilon)^{\beta}/\alpha\right],$$
(1)

where  $\varepsilon$ — parameter of a position of the distribution, when  $X \ge 0$   $\varepsilon = 0$ ;  $\alpha$  and  $\beta$ — the parameters of a scale and the form of distribution.

Let's dwell upon the normalized Veibull's distribution. The standardized variable  $\gamma = (X-X)/X$ , the expression  $X = X(\gamma V + I)$  will be used but  $\alpha$  should be neglected:

$$F(X) = 1 - \exp\left\{-\left[(\gamma V + 1)\Gamma(1 + \beta^{-1})\right]^{\beta}\right\},$$

$$f(\gamma) = \beta V \Gamma\left(1 + \beta^{-1}\right)^{\beta} \left(\gamma V + 1\right)^{\beta - 1} \exp\left\{-\left[(\gamma V + 1)\Gamma(1 + \beta^{-1})\right]^{\beta}\right\},$$
(2)

where  $\Gamma(1+\beta^{-1})$  — gamma-function.

As the coefficient of variation V remains constant the parameters of  $\beta$  and gamma-functions are not also changeable. So the approximate normalized presentation in the form of equation (2) remains unchangeable per year.

The analysis of the received results has demonstrated that the wind load is of a quasi-stationary origin with the stationary not only on frequency but on the normalized distribution of ordinate.

#### 3 PROBABILISTIC MODELS OF THE WIND LOAD

The systematic analysis of five most commonly used probabilistic presentations of random loads with the account of unspecified and normal laws of distribution were introduced in our previous work [2]. This article gives the general analysis, which is applied to the wind load described with Veibull's distribution, partially these results are represented in work [1]. For this model the solutions of the direct problem of the calculation of the wind level load  $\gamma(t)$  corresponding to the given probability Q(t) and opposite problem of the determination of Q(t) of exceeding  $\gamma$ -level during t were obtained. The evident advantage of all these solutions is the obvious account of a time factor "t".

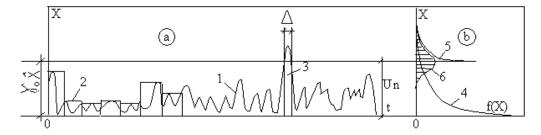


Fig. 3. Probabilistic models of the wind load: a — realization of load: 1 — random process; 2 — random sequence of independent loads; 3 — discrete presentation; b — density distribution: 4 — process ordinate; 5 — absolute maxima; 6 — extremes.

The main probabilistic model of wind load is the model presented in the form of a quasi-stationary random process, it is illustrated in figure 3, position 1. The number of outliers of this process gives the estimation Q(t):

$$Q(t) \cong N(\gamma, 0 \le \tau \le t) = 0.4 \omega t K_{tr} \beta \sqrt{V} \left[ \Gamma(1 + \beta^{-1})^{\beta} \left[ \gamma V + 1 \right]^{\beta - 0.5} \times \exp \left\{ - \left[ \left( \gamma V + 1 \right) \Gamma(1 + \beta^{-1}) \right]^{\beta} \right\}$$
(3)

where  $K_{tr}$  — the coefficient accounts the load wind trend during the year.

For the simplification of a design computation the parameter  $\delta$  is introduced, which numeric values are in the intervals of 0.54-0.66 and can be determined as follows:

$$\delta = -\ln\left[0.4\beta\sqrt{V}\Gamma(1+\beta^{-1})\right] - (\beta - 0.5)\ln(\gamma V + 1). \tag{4}$$

As a result the equation (3) will be transformed into

$$Q(t) = \omega t K_{tr} \exp \left\{ -\left[ (\gamma V + 1)\Gamma(1 + \beta^{-1})\right]^{\beta} - \delta \right\}.$$
 (5)

The model of the absolute maxima of random process treats the largest wind loads as random values. These loads are higher than characteristical maximum level

 $\gamma_0$  [5] as it's shown in figure 3, position 5. Its density distribution differs from Veibull's one:

$$f(\gamma) = \left(\frac{K_{tr}}{K_{tro}}\right) \left[\frac{V}{(\gamma V+1)}\right] \left\{\beta \left[\Gamma(1+\beta^{-1})(\gamma V+1)\right]^{\beta} - \beta + 0.5\right\} \times \exp\left\{\Gamma\left(1+\beta^{-1}\right)\left[(\gamma_{0}V+1)^{\beta} - (\gamma V+1)^{\beta}\right] + \delta_{0} - \delta\right\}$$
(6)

In this case the solution of a direct problem is as follows:

$$\gamma = \left\{ \left[ \left( \gamma_o V + 1 \right)^{\beta} + \left( \ln \left\{ \frac{K_{tr}}{Q(t)K_{tr}} \right\} \right) - \delta_o + \delta \right]^{\frac{1}{\beta}} - 1 \right\} V^{-1}, \tag{7}$$

where Ktro,  $\delta_0$  — refer to  $\gamma_0$  loadlevel.

Very often the wind loading is introduced in the form of a random sequence of independent loads as it's shown in figure 3, position 2. In this case the frequent parameter is the intensity  $\lambda$ , which equals the number of a load in per-unit time  $t_{\lambda}$ . The possibility of overloading of  $\gamma$  is:

$$Q(t) = \lambda t \exp\left\{-\left[\left(\gamma V + 1\right)\Gamma\left(1 + \beta^{-1}\right)\right]^{\beta}\right\}$$
 (8)

The probabilistic model of the wind load in the form of discrete presentation is depicted in fig. 3, position 3. It uses the time parameter-mean duration of overloading  $\Delta$ , connected with the intensity by the ratio  $\Delta = t_{\lambda}/\lambda$ ; in accordance with it the direct and opposit problems are solved like:

$$\gamma(t) = \left[ \left( \ln \left\{ t / \left[ Q(t) \overline{\Delta} \right] \right\} \right)^{1/\beta} \Gamma \left( 1 + \beta^{-1} \right) - 1 \right] V^{-1}; \tag{9}$$

$$Q(t) = \left(\frac{t}{\Delta}\right) \exp\left\{-\left[\left(\mathcal{W} + 1\right)\Gamma\left(1 + \beta^{-1}\right)\right]^{\beta}\right\}.$$
 (10)

In order to adopt the statistics of extremes [6], it's necessary to confirm that the wind load values samples which as presented in accordance with Veibull's distribution correspond to the exponential type. In this case the following condition should be satisfied:

$$\frac{f(X)}{[1-F(X)]} = -\frac{f'(X)}{f(X)} \tag{11}$$

If the Veibull's distribution function is performed in the form of (1) when  $\varepsilon=0$  the left part of equation (11) will be written as follows:

$$\frac{(\beta/\alpha)X^{\beta-1}\exp(-X^{\beta}/\alpha)}{\exp(-X^{\beta}/\alpha)} = \frac{\beta}{\alpha X^{\beta-1}}.$$

The derivative of Veibull's distribution density equals:

$$f'(X) = \left(\frac{\beta}{\alpha}\right) \exp\left(-\frac{X^{\beta}}{\alpha}\right) X^{2(\beta-1)} \left[ (\beta-1)X^{-\beta} - \frac{\beta}{\alpha} \right]$$

The first term in square brackets asymptotically approaches to zero with X growth. The right part of the condition (11) can be written like:

$$\frac{-\left(\frac{\beta}{\alpha}\right)^{2} \exp\left(-\frac{X^{\beta}}{\alpha}\right) X^{2(\beta-1)}}{\left(\frac{\beta}{\alpha}\right) X^{\beta-1} \exp\left(-\frac{X^{\beta}}{\alpha}\right)} = -\frac{\beta}{\alpha \beta^{-1}}.$$

Thus, the exponential condition of Veibull's sampling asymptotically performs. So the wind maximum values can be described correctly by the double exponential Gumball distribution of the I type [6], which is shown in figure 3, position 5. In this case the direct problem is solved with the help of the function from the volume of body sampling  $n_e$ :

$$\gamma(t) = \left\{ \ln n_{\rm e} \right)^{1/\beta} \left[ 1 - \ln Q(t) / (\beta \ln n_{\rm e}) \right] / \left( \Gamma (1 + \beta^{-1}) - 1 \right) V^{-1}.$$
 (12)

The formulae for necessary extremal parameter computation were represented in our paper [1].

#### 4 COMPARISON OF PROBABILISTIC MODELS OF WIND LOAD

All the examined models are close to its sense and forms of Q(t) evaluation:

$$Q(t) = \omega t K_{tr} f(\gamma) / \sqrt{2\pi} = f(\gamma) / f(\gamma_0) = \lambda t [1 - F(\gamma)] =$$

$$= t [1 - F(\gamma)] / \overline{\Delta} = 1 - \exp[-\exp(-\gamma)],$$
(13)

where  $F(\gamma)$  and  $f(\gamma)$  are integral and differential functions of a load distribution in accordance with the equation (2).

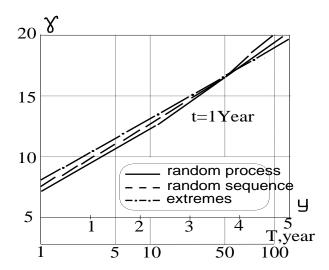


Fig. 4. The comparison of probabilistic models of the wind load on the extremal scale.

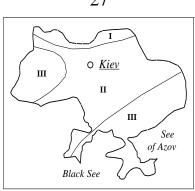


Fig. 5. The map of districts of Ukraine according to the wind pressure [7].

#### 5 MEAN WIND PARAMETERS OF UKRAINIAN DISTRICTS

Table 1. Probabilistic parameters of wind load

				Numeric values for wind districts of Ukraine		
No.	Model, parameter	Symbol	Unit			
				I	II	III
	Common parameters					
	Mathematic expectation	$\overline{X}$	kPa	8.6	14.6	22.3
	Coefficient of variation	V	-	1.81	1.78	1.73
	Veibull's form parameter	β	-	0.5862	0.5941	0.6078
	Gamma-function	$\Gamma(1+\beta^{-1})$	-	1.5518	1.5243	1.4796
1	Quasi-stationary random					
1	process					
			$\frac{1}{2}$ hours	Ţ		
	Effective frequency	$\omega$	24	6.58	5.16	5.42
2	Absolute maxima of random					
	process					
	Caracteristical maximum	$\gamma_0$	-	8.058	7.327	7.154
	level	. 0				
3	Random sequence		1			
	Load intensity	λ	year <sup>-1</sup>	650	480	465
4	Discrete presentation					
	Mean duration of overload-	$\overline{\Delta}$	hour	13.55	18.31	18.92
	ing	Δ	пош	13.33	10.51	10.72
5	Veibull's extremes per year					
	Body sampling	$n_e$	-	977	738	963
	Characteristical extremum	$u_n$	kPa	0.149	0.230	0.333
	Extreme intensity	$\alpha_{_n}$	$kPa^{-1}$	27.1	17.1	12.0

Provided with the equation (13) the formulae, which connect the parameters of different probabilistics models of the wind load were derived in our paper [1].

Load probabilistic comperison will be well performed at the extreme scale [6], which is illustrated in figure 4. On the axis of ordinate of a scale the standartized load

is laid off, on the axis of abscissae the Gumbel's distribution argument  $y = -\ln[-\ln F(t)]$  is laid off which is connected with the load return period T

Gumbel's distribution of the first type is discribed on the scale in the form of straight lines. The models of a random process, random sequences of independent loads and discrete representation are introduced as different curves.

In the case these curves are of a faint concaved character as it's shown in figure 4. The main advantage of this scale is its visual effects of the tail parts of the load distributions, which have rather small distinctions in the usual forms of presentation. It enables to present the visual comparison and correspondation of parameters of different load models.

In accordance with existing Codes [7] Ukraine is divided into three wind districts as it shown in figure 5. The II wind district occupies the main part of a country, the III district is situated in the south and south-east parts and some part of the Carpathian Mountains, the I district stretches along a narrow part of the north-west board of Ukraine.

The experimental wind probabilistic parameters of all examined models were approximated in accordance with the Ukrainian districts and tabulated in table 1. Some of these results were given in work [8]. The values  $\beta$ ,  $\Gamma(1+\beta^{-1})$  and coefficients of variations, which fully determine the average district Veibull's distributions of wind load derived from equation (2) are tabulated there. In table 1 the corresponding frequent characteristics are shown. The numeric values of parameters of different models were derived from the condition of close evaluation Q(t) for the working life t=50 years. This idea is well illustrated in figure 4.

#### 6 PROVISION OF DESIGN VALUES OF THE WIND LOAD

The results of the problem are tabulated in table 2. They demonstrated that specified and design values of the wind load correspond differently to the experimental statistic data for the Ukrainian territory. For the I-st district the design values are larger then the real ones, for the III district it is just on the contrary: the real loads can exceed the design ones. These data justify of the necessary of further researches and development of the National Codes of wind loads for Ukraine.

The worked out method, which examined the model and obtained parameters gives possibility to estimate reliability of a wide range structures under wind and other loading. Unfortunately the range of this paper doesn't allow representing all the results. We'd like to point out that the received data show that the reliability differs considerably for structures designed in accordance with the Building Codes [7]. This method also allows specifying some coefficients of design standards.

Table 2. Probabilistic provision of the design wind Loads on [7] for Ukraine

Wind Specified load Design load district  $w_m$ , Pa  $\gamma_n$   $T_n$ , years  $w_m \gamma_f$ , Pa  $\gamma_f$   $T_d$ , years

Wind	S	specified loa	.d	Design load			
district	$w_m$ , $Pa$	$\gamma_n$	$T_n$ , years	$w_m \gamma_f, Pa$	$\gamma_f$	$T_d$ , years	
I	230	14.22	11.6	322	20.13	86.8	
II	300	10.98	5.1	420	15.60	28.2	
III	380	9.27	2.5	532	13.21	13.4	

with  $w_m$  — specified mean wind load;  $w_m \gamma_f$  — the same design load;  $\gamma_f$  — load factor;  $\gamma_n = (w_m - X)/X$ ,  $\gamma_d = (w_m \gamma_f - X)/X$ ;  $T_n, T_d$  — standard deviation and the return period correspondingly to the specified design load.

#### 7 CONCLUSIONS

The large amount of statistics results on wind loads were examined for the territory of Ukraine. The wind load is of quasi-stationary origin with the constant frequent parameters and normalized distribution. For the description of ordinate density Veibull's law and double exponential Gumbel's distribution were used. The most widely spread probabilistic models of wind load were observed. They are as follows: quasi-stationary random process and its absolute maxima, random sequence of independent loads, discrete presentation and extreme model. Having integrated some initial data all necessary mean parameters of mentioned probabilistic wind models were determined for three districts of Ukraine. The worked out method and parameters give possibility to develop the probabilistic analysis of building structures.

#### 8 ACKNOWLEDGMENTS

The author wants to express his gratitude to associated professor V. A. Pashinski for his work on collecting and analyzing the wind load data.

#### 9 REFERENCES

- [1] Pashinski, V. A. and Pichugin, S. F. (1994). Wind Load Probabilistic Description and Value Computation Procedure Adopted for Building Code of Ukraine. Preprints EECWE'94; P.1, Vol.3, 49-52, Warsaw-Poland.
- [2] Pichugin, S. F. (1995). Probabilistic Presentation of Load Actings on Building Structures, News for High Educational Establishments, Construction, 4, 12-18, Novosibirsk, (in Russian).
- [3] Pichugin, S. F. (1995). Reliability Estimation of Steel Elements under Variable Loads, XLI Konf. Naukowa KILIW PAN i KN PZITB, Vol.3., 151-156, Krakow-Krynica, Poland.
- [4] Augusti, G., Baratta, A., Casciati, F. (1984). Probabilistic Methods in Structural Engineering. (Chapman and Hall), London, NewYork.
- [5] Loads and Loadings. (1987). Construction Standarts and Rules 2.01.07-85. Moscow, (in Russian).
- [6] Bolotin, V. V. (1969). Statistical Methods in Structural Mechanics, (Holden-Day), San Francisco.
- [7] Gumbel, E. J. (1967). Statistics of Extremes, (Columbia University Press), NewYork.
- [8] Pashinski, V. A. (1987). The Combination of Atmospheric and Crane Loads Applied to the Steel Frame of one Floor Industrial Buildings, Ph. D. Thesis, (Moscow Civil Engineering Institute), Moscow.

# PROBABILISTIC SPECIFICATION OF DESIGN WIND LOAD COEFFICIENTS

#### **SUMMARY**

Large number of wind meteorological data was gathered for the territory of Ukraine. Quasi-stationary model of the mean wind load was substantiated. Design coefficients for practical reliability computation were offered. Trend coefficient takes into account season changeability of wind load parameters. This coefficient together with the auxiliary one allows obtaining the closed solution for determination of wind load level, which corresponds, to a chosen probability of excess. A simple approximate formula for wind load values with different return period has been derived. Numeric values of temporary factor for practical calculations were obtained. It can be used for buildings with different terms of serviceability.

#### 1 INTRODUCTION

Mean wind load is the main one for the majority of buildings and structures. We consider this problem hasn't got enough estimation yet whereas the development of probabilistic description of the mean wind load is very important for the perfection of both Building Code design and the theory of building structure reliability. A large number of meteorological data was collected for the territory of Ukraine. Quasistationary model of the wind load was developed on its base [1,2]. For its practical computation usage the simplifications should be introduced in the form of designed coefficients. Return period of a wind load is being still ignored in structure design.

#### 2 TREND COEFFICIENT $K_{TR}$

The character of season change of wind load mathematic expectation  $\overline{X}$  for Ukraine is examined in the work [2]. Taking into account the mentioned above facts Weibull distribution for the mean wind load also depends on calendar time t:

$$F(x) = 1 - \exp\left[-x^{\beta}/\alpha(t)\right],\tag{1}$$

where  $\beta$  and  $\alpha(t)$  — parameters of a form and a scale of distribution correspondently.

From the developed probabilistic wind models the main one is the representation in the form of a random process. The formula of the number of outliers of a wind random process is in the form of:

$$v_{+}(x,t) = 0.4\omega[\beta/\alpha(t)]x^{\beta-0.5}[\hat{x}(t)]^{0.5} \exp[-x^{\beta}/\alpha(t)],$$
 (2)

where:  $\hat{x}(t)$  — wind standard deviation;  $\omega$  — constant effective frequency.

The estimation of probability of level excess is very important for reliability computation and requires the integration:

$$Q(x,t) = \int_{0}^{t} v_{+}(x,t)dt.$$
 (3)

For design simplification the coefficient of trend is introduced in the form of:

$$K_{tr} = \left[ \int_{0}^{t_{1}} v_{+}(x,t) dt \right] / \left[ v_{+}(x,t_{0}) t_{1} \right], \tag{4}$$

where:  $t_0 = 135$  days. It's a base date in January. It corresponds to the trend top of the mathematic expectation since the conventional date is the 1st of September;  $t_1$  — the design time interval which equals to 1 year.

Let's substitute the numeric integration to the summation of monthly outliers. Then the formula (3) will be as follows:

$$K_{tr} = \left\{ \sum_{i=1}^{12} \left( \alpha_0 / \alpha_i \right)^{1 - 0.5 / \beta} \exp \left[ -x^{\beta} \left( \alpha_i^{-1} - \alpha_0^{-1} \right) \right] \right\} / 12, \tag{5}$$

where:  $\alpha_i$  and  $\alpha_0$  - Weibull parameters of a scale for i month and the base one correspondently.

The obtained numeric values  $K_{tr}$  for the wind districts of Ukraine, which correspond to Building Codes [3] are tabulated in table 1 in the function of a return period of the wind load T. As it's presented in figure 1-a, the coefficient  $K_{tr}$  decreases if T is increased and it changes slightly in the interval  $T = 20 \div 50 \ years$ .

#### 3 AUXILIARY COEFFICIENT △

For the transition to a normalized Weibull distribution the normalized deviation  $\gamma = (x - \bar{x})/\hat{x}$  and the ratio  $\bar{x} = \alpha^{1/\beta} \Gamma(1 + \beta^{-1})$  ( $\Gamma(1 + \beta^{-1})$  — gamma function) were used. Then formula (1) is presented in the form of:

$$F(\gamma) = 1 - \exp\left\{-\left[(\gamma \mathcal{W} + 1)\Gamma(1 + \beta^{-1})\right]^{\beta}\right\},\tag{6}$$

Table 1. Numeric values of coefficients  $K_{tr}$ ,  $\delta$  and  $\gamma_T$ 

Coef- ficient	Wind district of Ukraine	1	2	5	10	20	30	50
	I	0.5205	0.4994	0.4749	0.4587	0.4434	0.4351	0.4257
$K_{tr}$	II	0.4920	0.4720	0.4490	0.4340	0.4200	0.4130	0.4040
	III	0.4406	0.4206	0.3980	0.3832	0.3698	0.3627	0.3541
	I	0.6591	0.6451	0.6280	0.6164	0.6042	0.5989	0.5918
δ	II	0.6508	0.6333	0.6144	0.6010	0.5888	0.5821	0.5739
	III	0.6224	0.6041	0.5824	0.5677	0.5540	0.5465	0.5373
γт	All districts	0.46	0.54	0.66	0.76	0.86	0.92	1.00

The same transformation is applied to formula (2) and with the account of  $K_{tr}$  for the evaluation (3) the formula (7) is derived:

$$Q(\gamma, t) = \omega t K_{tr} \exp(-\delta) \exp\left\{-\left[(\gamma V + 1)\Gamma(1 + \beta^{-1})\right]^{\beta}\right\}.$$
 (7)

In this formula the coefficient  $\delta$  equals:

$$\delta = -\ln \left[ 0.4\beta V^{0.5} (\gamma V + 1)^{\beta - 0.5} \Gamma (1 + \beta^{-1})^{\beta} \right]. \tag{8}$$

All the arguments of this formula are constant except  $\gamma$ . So the numeric values  $\delta$  for wind districts of Ukraine are in the narrow range of 0.54-0.66 for  $T=1 \div 50$  years (see table 1 and figure 1-b).

The use of coefficients  $K_{tr}$  and  $\delta$  reduces the design and excludes the numeric integration. As a result the closed solution of normalized load level has been derived which corresponds to a chosen excess probability Q(t):

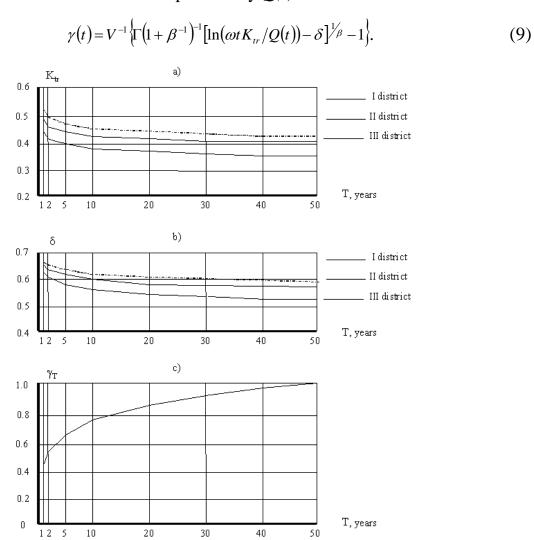


Fig. 1. Design coefficients of wind load in the function of its return period: a) trend coefficient  $K_{tr}$ ; b) auxiliary coefficient  $\delta$ ; c) temporary coefficient  $\gamma_T$ .

Average wind parameters  $\omega$ , V,  $\beta$ , and  $\Gamma(1+\beta^{-1})$  for Ukrainian territory are presented in the work [2]. The computation of normalized wind load values  $\gamma(T)$  which are connected with the return period T (in years) was significantly simplified with the

account of the mentioned above parameters and proposed coefficients  $K_{tr}$  and  $\delta$ . For this purpose the next simplified formula was offered:

$$\gamma(T) = a[\ln(bT)]^c, \tag{10}$$

where: a, b and c — numeric coefficients from table 2. The formula can be applied if T>20 years.

Wind district of Ukraine	a	b	c
I	0.356	556.1	1.706
II	0.369	428.5	1.684
III	0.391	409.2	1.647

Table 2. Coefficients of formula (10)

#### 4 TEMPORARY FACTOR $\gamma_T$

The obtained results allow to compute mean wind load which corresponds to different return period T:

$$w_m(T) = \bar{x}[1 + \gamma(T)V], \tag{11}$$

where:  $\bar{x}$ , V— mathematic expectation and wind load variation coefficient represented in [2];  $\chi(T)$  — standard deviation, derived from formula (9) or (10).

Temporary factor  $\gamma_T$  determined as (12) is recommended for practical application

$$\gamma_T = w_m(T)/w_m(T_0), \tag{12}$$

where:  $T_0$ =50 years — return period in accordance with the Building Code [3] for design load chose.

The corresponding arguments about  $\gamma_T$  for Ukrainian territory are given in table 1 and figure 1-c. Design wind load is multiplied by these coefficients in the computation of time strength of reconstructed members and some other objects with the limited term of serviceability. For the design of structures under erection it's allowed to apply specified wind load multiplied to the reduced coefficient  $\gamma_m = 0.55$ . This recommendation is based on the value  $\gamma_T$  for T=1 year.

#### 5 CONCLUSION

The results reported in this paper as well as ones presented in the previous author's works [1,2] suggest rather detailed probabilistic description of mean wind load for Ukraine. Proposed design coefficients significantly simplify the computation process and allow to obtain the wind load values in the form of closed ones for different return periods. A derived temporary coefficient  $\gamma_T$  allows increasing the structure economical efficiency under strength, reconstruction and erection. Numeric wind parameter values and design coefficients are developed for Ukraine. General approaches and design formulae are worked out and are of universal character and can be ap-

plied to any geographical districts. Probabilistic wind load model was successfully introduced into the computation of usual and specific structures [5,6].

#### 6 REFERENCES

- [1] Pashinski V. A., Pichugin S. F.: Wind Load Probabilistic Description and Value Computation Procedure Adopted for Building Code of Ukraine. In: Preprints EECWE'94, Warsaw, Poland, 4-8 July 1994, P.1, Vol. 3, p.p. 49-52.
- [2] Pichugin S. F.: Probabilistic Analysis on Wind Load and Reliability of Structures. In: Proc. 2EACWE, Genova, Italy, 22-26 June 1997, Vol. 2, p.p. 1883-1890.
- [3] Loads and Loadings (in Russian). Construction Standards and Rules 2.01.07-87, Moscow, 1987.
- [4] ENV 1991-2-4. Eurocode 1. Basis of Design and Actions on Structures, Part 2-4, Wind Actions. CEN, Brussels, 1994.
- [5] Pichugin S. F.: Reliability Estimation of Steel Elements under Variable Loads. In: XLI Konf. Naukowa KILIW PAN i KN PZITB, Krakow-Krynica, Poland, 1995, Vol. 3, p.p. 151-156.
- [6] Kondra M. P., Kopiiko O. V., Mikitarenko M. A., Perelmuter A. V. and Prusov V. A.: Integral Estimate of Risk under Wind Load upon Structures of the Encasement at Chernobyl Atomic Power Plant. In: Proc. 2EASWE, Genova, Italy, 22-26 June 1997, Vol. 2, p.p. 1833-1839.

# CERTAIN PROBLEMS IN PROBABILISTIC MODELLING OF WIND LOAD

Wind load probabilistic research for Ukraine is of great importance now. This is a problem, which deals with the necessity for developing National Codes of atmospheric loads on building structures as well as the need for reliability design of buildings and structures. This problem has been studied for several years at Poltava State Technical University. The propositions presented in this paper are the continuation of wind research results which were developed through the author's papers at 1<sup>st</sup> EECWE in Warsaw in 1994 [1], 2nd EECWE in Prague in 1998 [2] and 2nd EACWE in Genova in 1997 [3].

The mean and fluctuation values of wind load on a building structure are under investigation in this paper. The systematic results of mean velocity measurements at 70 Ukrainian meteorological stations had been taken as the initial ones. The mean wind force distribution and its numeric parameters were analysed with the help of a certain quadratic transformation of the Veibull's distribution of the wind velocity without the account of its direction.

The mean wind load is of a quasi-stationary origin. Its mathematical expectation and standard have a seasonal trend. This change is supposed to be accounted with a trend coefficient that is in limits of 0.35-0.40 the wind of Ukraine. At the same time wind frequent characteristics and normalised ordinate distribution remain constant during the season. For the description of mean wind ordinate density Veibull's

law and polynomial-exponential distribution were used. Wind load can be described faithfully by different probabilistic models: quasi-stationary differentiated random process with its absolute maxima, random sequence, discrete presentation and extremes. System comparative analysis of these wind load models was performed, and all necessary mean parameters of different probabilistic wind models were determined for three districts of Ukraine.

The fluctuation is presented in the form of a normal stationary random process. Its effective frequency was determined on the base of Davenport's spectrum.

On the base of calculation procedure of structure reliability under wind loading it's necessary to determine the sum or the differences of distributions using the formulae of convolution with numerical integration. Variants of convolutions of the normal law with the Veibull's distribution and the polinomial-exponential one were transformed, simplified and presented in the unified form suitable for comparison. The distribution form is presented as a function of normalized deviation from the centre with respect of a total standard of joint distribution. The joint distributions based on the proposed formulae can be non-symmetrical and differ considerably from the normal distribution in solutions of structural reliability problems.

Correlation functions and effective frequencies  $\omega$  of random processes of mean and fluctuation values of wind load differ considerably: for mean wind values  $\omega=5.4-6.6$  1/day, for fluctuation values  $\omega=4500-9000$  1/day. Therefore the joint action of these wind components become multi-frequent. This peculiarity can be defined by the structure complexity coefficient,  $\beta_{\omega}$  which equals the ratio of mean frequency on maximum to the effective frequency on zero of the random process.

The numerical modelling of wind stochastic process was realized in three phases. The following points were determined at the first phase:

- indispensable probabilistic characteristics;
- guess about a stationarity and ergodicity of wind stochastic process;
- adequate accuracy of probabilistic model;
- distribution laws of stochastic process ordinates;
- application of the correlation theory;
- minimal length of the realization of wind stochastic process (T = 1 year);
- necessary quantity of realizations for obtaining of the given model precision;
- quantity of a quantum on a level and time ( $\Delta t = 3$  hours).

At the following phase the general algorithm of model operation was developed and the numerical probabilistic model was worked out on the computer.

At the last phase the adequacy of wind stochastic model to actual wind process was estimated.

The numerical probabilistic model bases on the model of normalized stochastic stationary process with the Veibull's distribution of ordinates:

$$\xi_{(i)}(\gamma_{y}) = \frac{1}{V} \left\{ \frac{1}{\Gamma(1+\beta^{-1})} \left\{ -\ln[I] \right\}^{1/\beta} - 1 \right\},\tag{1}$$

where:

$$I = 1 - \frac{1}{\sqrt{2\pi}} \int_{a_L}^{\xi_{0(i)}(\gamma_x)} \exp[-0.5(\xi_{0(i)}(\gamma_x))^2] d\xi_{0(i)}(\gamma_x);$$

 $\beta$  is the parameter of the Veibull's distribution form;  $\Gamma(\bullet)$  is the Gamma function;  $a_L$  is the left-hand border of integration;  $\gamma_x$  is the normalized random value with a normal distribution law;  $\xi_{0(i)}(\gamma_x)$  is the ordinate of normalized normal stochastic process

$$\xi_{0(i)}(\gamma_x) = \sqrt{1 - [\exp(-\alpha_0 \, \Delta t)]^2} \gamma_x + [\exp(-\alpha_0 \, \Delta t)] \xi_{0(i-1)}$$
 (2)

with normalized correlation function

$$r(\tau) = \exp(-\alpha_0 |\tau|). \tag{3}$$

The ordinates of wind load stochastic process are determined as follows:

$$\xi_{(i)}(x;t) = [V \, \xi_{(i)}(\gamma_{y};t) + 1] \, \overline{X}(t), \tag{4}$$

where  $\overline{X}(t)$  is the trend function of wind load mathematical expectation.

These results were used for practical design of structure reliability. The worked out method, which examined the model and obtained parameters gives possibility to estimate reliability of a wide range structures under wind loading.

The analysis has demonstrated that steel elements under mean wind load designed in accordance with the existing code (glass elements, wind protection screens etc) are of sufficient reliability. So if the service term of these elements will be limited it is possible to introduce the temporary coefficient  $\gamma_T$ =0.46-1.0 for T=1-50 years for the design wind load. The obtained results allow decreasing considerably the design wind load for the conditions of structure erection. The consideration of the fluctuation wind component reduces seriously the level of structure reliability.

It is recommended to introduce the increasing coefficients of combination  $\psi = 0.7 - 0.9$  into the steel structure design under snow, wind and crane loads.

#### REFERENCES

- [1] Pashinski V. A. and Pichugin S. F. Wind Load Probabilistic Description and Value Computation Procedure Adopted for Building Code of Ukraine // Preprints EECWE`94. Warsaw Poland, 1994. p. 1, v. 3. p. 49 52.
- [2] Sergiy Pichugin. Probabilistic Specification of Design Wind Load Coefficients // 2nd East European Conference on Wind Engineering. Prague. Czech Republic. 1998. p. 511 515
- [3] Pichugin S. F. Probabilistic Analysis on Wind Load and Reliability of Structures// Proceedings of the 2 EACWE. Geneva. Italy. 1997. vol. 2. p. 1883 1890.

SNOW LOAD ON BUILDING STRUCTURES

# ABILISTIC DESCRIPTION OF GROUND SNOW LOAD FOR UKRAINE

### **SUMMARY**

This material is based on a large bulk of meteorological data. Considerable probabilistic characteristics of ground snow load were revealed through its study. Ukrainian winters are changeable with little too much snow. Various models of snow loads were developed. Method of parameter comparisons of different snow models was worked out. All necessary probabilistic parameters were obtained for snow districts of Ukraine. It is demonstrated that the design snow loads are considerably lower than the actual ones. It's pointed out that the development of National Codes of atmospheric loads can be considered as one of the main tasks.

### 1 INTRODUCTION

Snow load probabilistic research for Ukraine is of great importance now. This is a problem which deals with the necessity for developing National Codes of atmospheric loads on building structures as well as the need for reliability design of buildings and structures, preventing structure failures which routinely occur every winter in Ukraine. Snow loads are described with different probabilistic models by many authors. However, these approaches lack both systematic analysis and model comparison which can cause different results in structure reliability design. This problem has been studied for several years at Poltava State Technical University. The results presented in this paper are the integral part of a structure reliability estimation method developed through the author's work (Pichugin 1995a,b).

## 2 SNOW LOAD PROBABILISTIC DISTINCTIONS

The results of regular snow measurements for 15–40 years at 62 Ukrainian meteorological stations have been taken as the reference statistic material. Ground snow load realisations were obtained, their intervals run 5 or 10 whole days. This statistic analysis has demonstrated some specific characteristics of snow loading in the territory of Ukraine.

## 2.1 Snow season cycle

During winter, snow load has two little transitional irregular parts. The beginning of winter is the period of snow accumulation, the end of winter is the snow melting stage. The main winter period lies between the winter average beginning date  $t_S$  and the average end date  $t_F$ . The 10th–15th of November is the starting point of winter for Ukraine ( $t_S$ ) and the 10th of April can be treated as the end of it ( $t_F$ ). The period  $t_W = t_F - t_S$  is the most important stage of stable snow cover. At this time snow load has relatively high values, which are of interest to the structure reliability design. Within this period ( $t_W$ ) some general rules of snow load can be traced and they will be described later.

## 2.2 Season winter period

Seasonal changes of mathematical expectation  $\overline{X}(t)$  and standard  $\hat{X}(t)$  of snow load (t - is) the time interval which is calculated from the 1st of September) is of skewness nature and its top corresponds to the middle of February (Fig. 1). This trend is described approximately as a polynomial of the 3rd degree. Meanwhile, the district coefficients of variation V and relative skewness S can be roughly treated as constants (Pachinski 1999). In this paper we use the snow loads for different districts of Ukraine in accordance with the Ukrainian Code (Loads 1987).

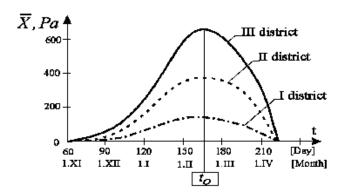


Fig. 1. Season change of the snow load mathematical expectation.

## 2.3 Frequent character of snow loads

Stable snow load (in the interval  $t_W = t_F - t_S$ ) is of a stationary frequent character. This conclusion is based on the fact that snow normalised correlation function and effective frequency have no significant distinction during winter.

### 2.4 Distribution of snow load values

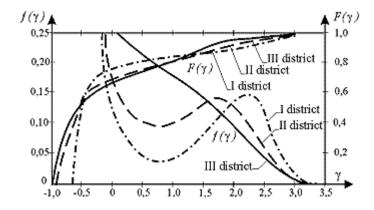


Fig. 2. Normalised district snow load distributions.

Experimental snow distributions for Ukrainian territory are of bimodal character. Therefore (Fig. 2), the normal low that satisfactory describes the snow loads of many snow districts cannot be applied in these cases. That's why so-called polynomial-exponential distribution was substantiated and used. Its normalised presentation is as follows:

$$f(\gamma) = \exp(C_0 + C_1 \gamma + C_2 \gamma^2 + C_3 \gamma^3), \tag{1}$$

where  $\gamma = (x - \overline{X})/\hat{X}$  — normalised deviation of snow load.

If V and S are constant the coefficient of exponential index in expression (1) remains constant and does not depend upon the date (t). As a result snow load is stationary not only in frequency, but in normalised distribution of ordinate (1).

## 2.5 Specific characters of snow polynomial-exponential distribution

This distribution (Fig. 2) has an exponential maximum to the left at the original coordinates, which means absence of snow during some winter periods. This is a specific characteristic of changeable Ukrainian winters with little snow. The second top of the curve is determined by the period of stable snow loads. It is interesting to analyse the influence on distribution of coefficients of the polynomial argument in expression (1). Term  $C_0$  determines the original ordinate,  $C_1$  determines down — going exponential part, positive coefficient  $C_2$  — existence and height of a curve top. The last negative multiplier,  $C_3$ , suppresses the effect of  $C_2$  to the level of  $\gamma = (3-3.5)$  and it makes a curve tag down to the X-axis. Thus, it is not by coincidence that the distribution tags (1) are located lower then the normal distribution ones. It is necessary to note that charts of integral functions of snow load distribution  $F(\gamma)$ , are of more stable and smooth character in comparison with differential functions  $f(\gamma)$  (Fig. 2).

## 3 SNOW LOAD AS A RANDOM PROCESS

## 3.1 Outliers of snow random process

Mentioned above, probabilistic specific characteristics were taken into account while presenting ground snow load in the form of quasi-stationary differentiated random process. Mathematical expectation,  $\bar{X}(t)$ , and the standard,  $\hat{X}(t)$ , of the process change during a seasonal cycle just as the effective frequency,  $\omega$ , and normalised distribution of ordinate (1) remain constant. The outlier frequency of this process for the moment (t) of a seasonal cycle can be derived from a stationary process (Pichugin 1995a)

$$v_{+}(x,t) = \omega f(\gamma) / \sqrt{2\pi} = \omega \exp(C_0 + C_1 \gamma + C_2 \gamma^2 + C_3 \gamma^3) / \sqrt{2\pi},$$
 (2)

where  $x = \overline{X}(t)(1+\gamma V)$  is the chosen level of snow load;  $\gamma = [x - \overline{X}(t)]/[\overline{X}(t)V]$  — normalised load deviation.

Let us estimate the probability of snow load excess of level x per year (opposite problem). It is determined by the quantity of outliers of quasi-stationary random process over that level. It is calculated with the help of the integral expression (2) at the interval  $t_W$ 

$$Q(x,t=1 y ear) = N_{+}(x,t_{w}) = \int_{0}^{t_{w}} v_{+}(x,t) dx.$$
 (3)

The mean annual curve of snow load outliers for different districts was obtained by summation of numbers of outliers for the fortnight intervals. These curves are presented in Figure 3. They are characterised by sharp peaks at the beginning and

long extended parts at the level of  $N_+$  (1 year)=0.2–1.0. These parts are formed by ground snow storage and the snow-melting period during different months. This specific characteristic is typical for rather warm Ukrainian winters. The nature of outlier curves has no simple analytical description, but it is much simpler than the distribution (1).

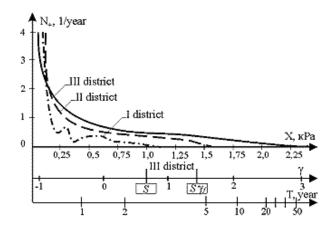


Fig. 3. Number of snow load outliers (per year).

Let us take into account the proportion between the number of outliers per year and return period  $T=1/N_+$  (1 year). Than, it is possible to determine the loads of different return periods directly using the outlier curves. These loads are shown on the additional low scale in Figure 3. Because of specific characteristics of snow loads an outlier comparatively low normalised snow load value  $\gamma = 0.2-0.4$  correspond to the return period T (which equals 1 year). These values are located at the beginning of the rise. The transition to T period, which is 5–10 years, occurs as the shift along the stretch curves to level with  $\gamma = 1.5-2.7$ . The loads of higher return periods (T=20-50 years) are closely grouped, located on descending outlier curve parts  $\gamma = 2.5-3.0$ .

## 3.2 Trend coefficient

The trend coefficient,  $K_{tr}$ , was introduced to simplify computation and to get closed decisions. This coefficient equals the proportion of numbers of quasistationary random process outliers during the  $t_W$  period to the number of outliers during the same stationery process period. It corresponds to the trend top for  $t_0$ =165 whole days:

$$K_{tr} = \frac{\int_{0}^{t_{W}} v_{+}(x,t) dt}{v_{+}(x,t_{0})t_{w}}.$$
 (4)

Figure 4 illustrates the district values  $K_{tr}$ . They are in the intervals between 0.13–0.70 for T=1-50 years. If T>10 years, they slowly descend. Instead of expression 3 for probabilistic description of snow load (inverse problem),  $K_{tr}$  application allows one to use the dependence for stationary process in the form of

$$Q(x,t) = \frac{tt_w K_{tr}}{\sqrt{2\pi}} \exp(C_0 + C_1 \gamma + C_2 \gamma^2 + C_3 \gamma^3),$$
 (5)

where  $\gamma = [x - \overline{X}(t_0)]/[\hat{X}(t_0)]$ ; t — is the serviceability term in years.

This approach allows, one to derive the normalised level of snow load  $\gamma$  (from 5). It corresponds to a given probability of exceeding [Q(t)] (direct problem). This decision is a root of a cube equation

$$C_{1}\gamma + C_{2}\gamma^{2} + C_{3}\gamma^{3} + \left\{C_{0} + \ln\left[\frac{tt_{w}K_{tr}\omega}{Q(t)\sqrt{2\pi}}\right]\right\} = 0.$$
 (6)

### 4 SNOW LOAD PROBABILISTIC MODELS

## 4.1 Absolute maxima of a random snow process

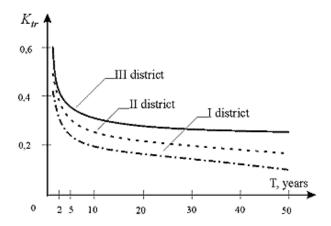


Fig. 4. Trend coefficient of snow load

As mentioned above, models in the form of quasi-stationary random process fully characterise ground snow load, but it takes a lot of initial statistical information which is difficult to access. The model, in the form of random snow load maxima, is more laconic and accessible. Distribution of maxima determined by the tag part of this random process is located higher than the level of characteristical normalised maximum  $\gamma_0$ . The letter is derived from the equation  $N_+(\gamma_0;0 \le \tau \le t)=1$ , where  $N_+(\bullet)$  — the number of outliers of random process (Bolotin 1969). This distribution for absolute snow maxima has the integral and differential function:

$$F(\gamma, \gamma_0) = 1 - \frac{K_{tr}}{K_{tr0}} \exp[C_1(\gamma - \gamma_0) + C_2(\gamma^2 - \gamma_0^2) + C_3(\gamma^3 - \gamma_0^3)], \tag{7}$$

$$f(\gamma;\gamma_0) = \frac{K_{tr}}{K_{tr0}} \left( C_1 + 2C_2 \gamma + 3C_3 \gamma^2 \right) \times \exp \left[ C_1 (\gamma - \gamma_0) + C_2 (\gamma^2 - {\gamma_0}^2) + C_3 (\gamma^3 - {\gamma_0}^3) \right], \tag{8}$$

where  $K_{tr}$  and  $K_{tr0}$  are trend coefficients corresponding to the  $\gamma$  and  $\gamma_0$  levels. It's easy to see that the distribution (8) is normalised (neglecting  $K_{tr}/K_{tr0}$ ). The example of a distribution like that is given in Figure 5.

## 4.2 Random sequence and discrete presentation on snow load

Snow load is presented rather often as a random sequence of independent loads. This model is sometimes called a scheme of independent tests. The load intensity,  $\lambda = N/t_{\lambda}$ , is the frequent characteristic here, N is the number of loads;  $t_{\lambda}$  is the chosen time interval. The answer to inverse problem is derived from the use of the integral function  $F(\gamma)$ 

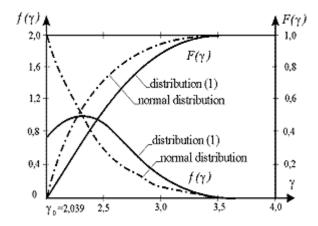


Fig. 5. Absolute maxima of snow load random process (II district, t=5 years).

$$Q(\gamma,t) = \lambda t \left[1 - F(\gamma)\right] = \lambda t \int_{\gamma}^{\infty} \exp\left(C_0 + C_1 \gamma + C_2 \gamma^2 + C_3 \gamma^3\right) d\gamma.$$
 (9)

## 4.3 Direct problem can be reduced to the solution of a cube equation

$$C_0 + C_1 \gamma + C_2 \gamma^2 + C_3 \gamma^3 + \ln \left[ \frac{\lambda t}{\mu(\gamma) Q(t)} \right] = 0,$$
 (10)

where  $\mu(\gamma) = f(\gamma)/[1 - F(\gamma)]$  — distribution intensity of a snow load.

Applying to snow load, this model advantage is the possibility to obtain a priori  $\lambda$  parameter (for example for month or annual intervals) and relative access of meteorological data in the intervals mentioned above. Besides, this integral function  $F(\gamma)$  is of a smoother character and is more suitable for computation in comparison to the differential function  $f(\gamma)$  (Fig. 2). Discrete presentation of snow load is of the same form, its frequent parameter is mean duration of overloading  $\overline{\Delta}(\gamma)$ . In this case the design formula are derived from (9) and (10) with the help of substitution  $\lambda = \overline{\Delta}^{-1}$ .

### 4.4 Snow extremes

This presentation is widely used and it describes well annual snow maxima. Gumball's well known distribution of the I-st type (Gumball 1967) is used

$$F(y) = \exp[-\exp(-y)], y = \alpha_n(\gamma - u_n), \tag{11}$$

where  $u_n$  and  $\alpha_n$  are the characteristically extremum and experimental intensity correspondingly, which depend upon body sampling  $n_e$ .

Let's check the possibility of applying experimental presentation for maxima from snow load samples, subjected to polynomial and exponential distribution (1). With this purpose, we'll check the validity of condition:

$$\mu(\gamma) = f(\gamma)/[1 - F(\gamma)] = \mu'(\gamma) = -f'(\gamma)/f(\gamma).$$

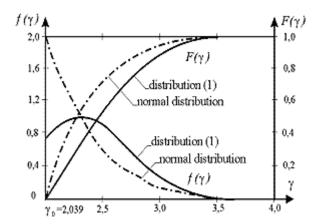


Fig. 6. Estimation of the exponential condition for snow body sampling (II district)

Its left part is evaluated numerically; the right part is of closed type  $C_1 + 2C_2\gamma + 3C_3\gamma^2$ . As it is possible to see at Figure 6, the given condition is carried out from the level  $(3.5-4)\gamma$ . Thus, snow load sample is of an exponential type and the extremum use is quite correct in that case.

For selection procedure simplification of  $u_n$  let's connect general extreme proportions with snow load distribution

$$Q(u_n) = n_e^{-1} = f(u_n) / \mu(u_n) = \exp(C_0 + C_1 u_n + C_2 u_n^2 + C_3 u_n^3) / \mu(u_n).$$
 (12)

Cube equation for  $u_n$  estimation is derived from (12)

$$C_0 + C_1 u_n + C_2 u_n^2 + C_3 u_n^3 + \ln[n_e / \mu(u_n)] = 0.$$
 (13)

If relation  $u_n = \gamma - y/\alpha_n$  is substituted into (12) we will obtain expression (14) for the body sampling (by  $\alpha_n$ ).

$$n_e = \alpha_n \exp\left[-C_0 - C_1(\gamma - y/\alpha_n) - C_2(\gamma - y/\alpha_n)^2 - C_3(\gamma - y/\alpha_n)^3\right]. \tag{14}$$

Another expression for  $n_e$  is derived from (12) by  $u_n$ 

$$n_e = \mu(u_n) \exp(-C_0 + C_1 u_n - C_2 u_n^2 - C_3 u_n^3). \tag{15}$$

## 5 COMPARISON OF PROBABILISTIC SNOW MODELS

The connection of models in the form of random process, random sequence, and discrete presentation is described as follows

$$\lambda = \Delta^{-1} = \omega \mu(\gamma) K_{tr} / \sqrt{2\pi} . \tag{16}$$

The selection of corresponding extreme parameters can be performed numerically or with the help of extreme scale (Fig. 7). Normalised deviation of  $\gamma$  load is graphed on the ordinate axis and distribution argument (11)  $y = -\ln[-\ln F(t)]$  or corresponding probabilities Q(t) of  $\gamma$  level exceeding connected with the return period of T load is graphed on the abscissa axis. Random process transition to the scale (Fig. 7) is carried out by annual outliers, the number of which equals  $N_+(\gamma,1)$ 

$$y = -\ln N_{+}(\gamma, 1year). \tag{17}$$

As it can be seen in Figure 7, a random process chart is of a complicated and irregular character if T < 5 years. The curves which illustrate random sequences and discrete presentations are of a smoother character and approach to random process curves at the top quite closely if  $T \ge 5$  years. Straight lines in Figure 7 show gumball's extremes. The analysis demonstrated that the most suitable snow extremes are from the 10-year samples. The corresponding line is tangent to the random process curve at point T = 10 years, it exceeds, insignificantly, snow loads for T = 10 - 50 years and it is higher than the irregular part of the curve for T < 10 years, which is of no importance.

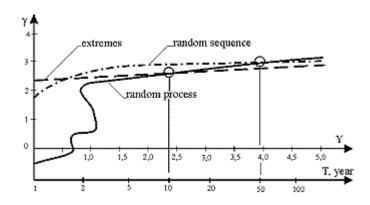


Fig. 7. Exposition of snow load models at the extreme scale

## 6 MEAN SNOW LOAD PARAMETERS OF UKRAINIAN DISTRICTS

All necessary mean snow load parameters were determined, for the mentioned above probabilistic models of ground snow loads, for districts of Ukraine (Table 1). In accordance with existing Codes (Loads 1987) the territory of Ukraine can be divided into three snow districts. The first district stretches along southern and western Ukrainian parts; the second one is situated in the northern and western parts of Ukraine and the third one is the narrow area in the northeast territory. Analytical expressions, numeric methods, and extreme scales (Fig. 7) were used. Snow load probabilistic model parameters were determined on the basis of equality of normalised load level  $\gamma$  (T=50 years) (Fig. 7).

There are, in Table 1, the largest values of mathematical expectation for all examined snow load models corresponding to the top of a seasonal trend, coefficient of variations and coefficients of polynomial-exponential distribution of the ordinate (1). In addition, some particular characteristics for every model are tabulated there (Table 1). These data allow one to give a full description of every possible snow model. It is

necessary to pay attention to the fact that all the models give close results in normalising snow load and structure reliability estimation. The author describes wind load in the same way (Pichugin 1997).

Table 1. Probabilistic parameters of snow load

	Numeric values for snow districts of Ukraine					
Model, parameter, symbol, unit						
	I	II	III			
Common parameters						
Mathematics expectation, $\bar{X}$ , kPa	0.164	0.344	0.631			
Coefficient of variation, V	1.60	1.26	0.920			
Coefficients of equation (1)						
$C_0$	-2.265	-1.736	-1.313			
$C_1$	-3.885	-1.926	-0.725			
$C_2$	3.855	1.885	0.445			
$C_3$	-0.920	-0.506	-0.178			
Quasi-stationary random process						
Effective frequency $\omega$ , 1/24 hours	0.141	0.095	0.073			
Absolute maxima of random process						
Characteristically maximum level, $\gamma_0$	2.688	2.459	2.113			
Random sequence						
Load intensity, $\lambda$ , year <sup>-1</sup>	7.90	4.86	3.20			
Discrete presentation						
Mean duration of overloading, $\overline{\Delta}$ , hour	13.55	18.31	18.92			
Extremes per year / per 10 years						
Pody sampling n	7.14	3.98	2.42			
Body sampling, n <sub>e</sub>	71.4	$\overline{40.0}$	$\overline{24.2}$			
Characteristically extremum, u <sub>n</sub> , kPa	0.738	1.074	1.163			
	0.86	1.409	1.858			
Extreme intensity, $\alpha_n$ , kPa <sup>-1</sup>	17.883	6.860	3.310			

### 7 PROVISION OF DESIGN VALUES OF GROUND SNOW LOAD

Generalised snow load parameters are tabulated in table 1 and they make it possible to vary the use of probabilistic structure design. In particular, using the obtained results, it was possible to estimate quantitatively the existing Codes of snow load for Ukrainian territory. The obtained results are given in Table 2 where S is specified snow load;  $S_{\gamma f}$  design snow load;  $\gamma_f$  load factor;  $\gamma_n = (S - \overline{X})/\hat{X}$  standard deviation of standard load;  $\gamma_d = (S\gamma_f - \overline{X})/\hat{X}$  the same of design load;  $T_n$ ,  $T_d$  standard deviation and return period correspondingly to the specified and design snow load.

The data given in Table 2 demonstrate that specified and design snow loads are of short return periods like T=2.53-3.85 years. It justifies that design loads in accordance with the Codes (Loads 1987) are not ensured enough and are much lower than

real ground snow loads for Ukrainian territories. Everything mentioned above gives evidence that the development of a National Ukrainian Snow Load Code is an urgent task. This fact was pointed out by the author in his previous works (Pichugin 1994). As a temporary measure the increasing of snow load factors can be proposed i.e. from  $\gamma_f = 1.4$  and 1.6 to  $\gamma_f = 2.4-3.0$  (Pichugin 1994).

Snow Load value S, Pa  $T_n$ , years  $\gamma_n$ district Specified I 500 1.29 2.72 load II 700 0.82 2.94 III 1000 0.64 2.53  $S_{\gamma f}$ . Pa  $T_d$ , years  $\gamma_d$ 700 2.43 Ι 2.06 Design load II 980 1.47 3.15

Table 2. Probabilistic provision of the design snow load for Ukraine

## 8 CONCLUSION

1.26

3.85

1400

Ш

Ground snow load is of a quasi-stationary origin. Its mathematical expectation and standard have a seasonal trend. At the same time snow frequent characteristics and normalised ordinate distribution remain constant during the season. Different probabilistic models can describe Snow load faithfully: quasi-stationery differentiated random process with its absolute maxima, random sequence, discrete presentation, and extremes. System comparative analysis of these snow load models was performed, and parameters for Ukrainian districts were validated. These results can be used for practical design of structure reliability. It is substantiated that existing Codes considerably underestimate real snow loads and they are badly in need of updating.

#### 9 REFERENCES

- [1] Bolotin, V. V. 1969. Statistic Methods in Structural Mechanic. San Francisco: Holden Day.
- [2] Gumball, E. I. 1967. Statistics of Extremes. New York: Columbia University Press.
- [3] Loads and Loadings, 1987. Construction Standarts and Rules 2.01.07-85. Moscow: Stroyisdat (in Russian).
- [4] Pashinski, V. A. 1999. Building Structures under Atmospheric Loads in Ukraine. Kiev: UkrNIIProectstalconstructsia.
- [5] Pichugin, S. F. 1994. Design Coefficients of Codes Based on the Reliability Analysis of Steel Structures. Ukrainian Construction (1): 18-20.
- [6] Pichugin, S. F. 1995a. Probabilistic Presentation of Load Actings on Building Structures. News for High Educational Establishments. Construction (4): 12-18 (in Russian).

- [7] Pichugin, S. F. 1995b. Reliability Estimation of Steel Elements under Variable Loads. XLI Konf. Naukowa KILIW PAN i KN PZITB "Krynica 1995". Krakow Krynica: Poletechnica Krakowska.
- [8] Pichugin, S. F. 1997. Probabilistic Analysis on Wind Load and Reliability of Structures. Proc. 2nd European & African Conf. Wind Eng., Geneva, Italy, June 22-26 1997. Padova: SGFditoriali.

# RELIABILITY OF STRUCTURES UNDER SNOW LOAD IN UKRAINE

#### **SUMMARY**

A model of quasi-stationary stochastic process with generalised year distribution density for ground snow load was out for the climatic conditions of Ukraine. Numeric probabilistic models of snow load were developed for personal computer realisation. Algorithm of steel structures probability estimation under snow load was done by different probabilistic techniques. This method has been realised in MathCAD mathematics programming environment by special program application complex. Large scale mathematical experiment for computation of steel structures reliability under snow load was performed. The reliability of steel structures (beams, trusses) under snow load was estimated by a developed method. These structures are of different mass roofs and snow loads for all Ukrainian districts. The calculation demonstrated the lack of reliability of rafter structures. That justifies the idea of understating of existing snow loads Code in Ukraine. Besides this fact validates the causes of steel truss failures. It applies to steel structures with light weight roofs in the southern districts of Ukraine when there is much snow in winter. The increasing of design snow load in 1.5-2 times for Ukraine region can solve the reliability problem of steel structures under snow load. Finally the groups of non-reliable steel structures have been defined to border normative design and develop similar-reliability design methods.

### 1 INTRODUCTION

For the last 10-15 years the changes of climatic conditions in the territory of Europe have been watching. The climatic changes have also tested at the territory of Ukraine. So the nature of operating snow loads changes: precipitation becomes short-lived, but intensive. The increase of the snow load for short period in a number of cases results to failures of building structures. At the present time it is important to evaluate the reliability of existing buildings and to find out potentially dangerous groups of structures for their precise calculation. Hence it is necessary to get a model of a snow load with provision for all particularities of its stochastic nature. It is necessary also to develop the general strategy of evaluation of structure reliability under the action of a snow load. This work continues the snow probabilistic investigations which were exposed during the 4-th International Conference on SNOW ENGINEERING, Trondheim, Norway, and June 19-22, 2000 (Pichugin 2000).

## 2 NUMERICAL PROBABILISTIC MODEL OF GROUND SNOW LOAD

In order to obtain a detailed analysis of snow load it is important to get a numerical probabilistic model for personal computer realisation. Acceptance of such model and its numerical modelling are being run for three stages.

## 2.1 Starting-up stage

At the given stage:

- the list of the probabilistic characteristics for creation of numerical probabilistic model of load is established;
- the stationary and ergodicity of load process is studied;
- the adequate and sufficient accuracy of load model is fixed. The probability of non-exceeding of a limit error is specified for the evaluation of the probabilistic characteristics. One order of error is established for all stages of numerical simulation;
- the ordinate distribution law of the load process is determined;
- the application of the correlation theory is substantiated;
- the minimum length of one realisation is identified. Sufficient quantity of realisations for obtaining given accuracy of model process is determined;
- the quantum value step in view of a level and time is determined for adequate accuracy of researches.

Today the results of regular snow measurements are used as the basic source of experimental data for ground snow load in Ukraine. A conducted analysis of experimental results of snow load observations was established in papers (Pichugin 1998, 2000). Let's define the following particularities of load as:

- the regular observations out a snow cover of the territory of Ukraine are conducted once in 5 whole days;
- the ground snow load is quasi-stationary stochastic process with the annual seasonal cycle (mathematical expectation and standard have an annual trend; a coefficient of variation, normalised correlation function (NCF) and effective frequency remain constant during the year);
- the ordinates of snow load stochastic process are described with a polynomial-exponential distribution law:

$$f(x) = \exp\left[\sum_{k=0}^{z} (C_k x^k)\right],\tag{1}$$

where z — the degree of a polynomial (for the territory of Ukraine z=3);  $C_k$  — factors of a polynomial (k=0...z);

• the normalised correlation function is described by the expression

$$r(\tau) = \exp(-\alpha \tau), \tag{2}$$

where  $\alpha$  — the parameter of NCF;

• the following length of realisation is received for snow load simulation

$$T_W = t_E - t_S, \tag{3}$$

where  $t_s$  and  $t_E$  — are mean date of the beginning and the end of winter, correspondingly.

## 2.2 Development of general algorithm of snow load simulation, creation of numerical model for personal computer

It is necessary to form realisations of large length for simulation of loads during the service life of structures. The methods of simulation of processes for given correlation functions and one-dimensional distribution laws are the most suitable (Bykov 1971). The given problem is solved by specially selected non-linear transformations of the corresponding normal stochastic processes.

Let's consider the simulation of normalised polynomial-exponential stationary stochastic process  $\xi(t)$  with a normalised correlation function  $r(\tau)$ . Integral and differential functions of the given distribution will look like the following:

$$F(\gamma_{y}) = \int_{a_{L}}^{\gamma_{y}} \exp(C_{0} + C_{1}\gamma_{y} + C_{2}\gamma_{y}^{2} + C_{3}\gamma_{y}^{3}) d\gamma_{y},$$
 (4)

$$f(\gamma_{v}) = \exp(C_0 + C_1 \gamma_{v} + C_2 \gamma_{v}^2 + C_3 \gamma_{v}^3),$$
 (5)

where  $a_L$  — a left border of distribution.

The function of transformation  $\gamma_{v} = f(\gamma_{x})$  is:

$$\int_{a_{I}}^{\gamma_{y}} \exp(C_{0} + C_{1}\gamma_{y} + C_{2}\gamma_{y}^{2} + C_{3}\gamma_{y}^{3}) d\gamma_{y} = \frac{1}{\sqrt{2\pi}} \int_{b_{I}}^{\gamma_{x}} \exp\left(-\frac{\gamma_{x}^{2}}{2}\right) d\gamma_{x}.$$
 (6)

The equation (6) has no an analytical solution. So it is solved numerically. For this purpose the density function of a normalised normal distribution is changed by the cut function in some interval  $[b_L; b_R]$ . The probability  $P_b$  of an output for borders of given interval is very small and it depends of a given accuracy of calculations. For example, at value  $b_L = -4.0 - P_b = 6.334 \cdot 10^{-5}$ , and at  $b_L = -5.0 - P_b = 5.733 \cdot 10^{-7}$ . We set by discrete values  $\gamma_x$  in an interval  $[b_L; b_R]$  and we shall receive the corresponding values  $\gamma_y$ . Thus, the function  $\gamma_y = f(\gamma_x)$  is presented in a tabular form. It is not quite suitable for the further usage. We approximated the relation  $\gamma_y = f(\gamma_x)$  by a function of a type like:

$$\gamma_{y} = f(\gamma_{x}) = P_{0} + P_{1}\gamma_{x} + P_{2}\gamma_{x}^{2} + P_{3}\gamma_{x}^{3} + P_{4}\gamma_{x}^{4} + P_{5}\gamma_{x}^{5}, \tag{7}$$

where  $P_i$  (i=0...5) — factors of a polynomial which are determined by the method of the smallest squares.

The modelling of normalised polynomial-exponential stationary stochastic process  $\xi(t)$  with a normalised correlation function  $r(\tau)$  is reduced to the formation of

discrete realisation  $\xi_0[n]$  of the normalised normal stochastic process  $\xi_0[t]$  and its transformation in the formula:

$$\xi(\gamma_{y}) = P_{0} + P_{1}[\xi_{0}(\gamma_{x})] + P_{2}[\xi_{0}(\gamma_{x})]^{2} + P_{3}[\xi_{0}(\gamma_{x})]^{3} + P_{4}[\xi_{0}(\gamma_{x})]^{4} + P_{5}[\xi_{0}(\gamma_{x})]^{5},$$
(8)

Here is the numerical model of normalised normal stochastic process with NCF (2):

$$\xi_{0(i)}[\gamma_x] = \sqrt{1 - \rho^2} \, \gamma_x + \rho \, \xi_{0(i-1)}[\gamma_x], \tag{9}$$

where  $\xi_{0(i)}[\gamma_x]$  — an ordinate of normalised normal stochastic process,  $\rho = \exp(-\gamma^*)$ ,  $\gamma^* = \alpha \Delta t$ ,  $\gamma_x$  — normalised normally distributed random variable (mathematical expectation is  $\overline{X} = 0$ , standard is  $\hat{X} = 1$ ,  $\Delta t$  — a step of a discretisation of a random load (the interval between adjacent observations).

The modelling results of normalised polynomial-exponential stationary stochastic process are given in Figure 1.

## 2.3 Snow load model for the territory of Ukraine

This model is based on the introduced above formula (8). Factors to a polynomial (8) for snow regions of Ukraine are specified in Table 1.

The transition from the normalised form) to the stochastic process with real distribution of ordinates is defined by the formula:

$$\xi_{(i)}(x;t) = \left[V\xi_{(i)}(\gamma_{y};t) + 1\right]\overline{X}(t), \tag{10}$$

where V — the coefficient of snow load variation;  $\overline{X}(t)$  — the trend function of a snow load mathematical expectation. A random value of snow load is always positive, that is  $\xi(x;t) \ge 0$ , therefore  $\gamma_y \ge -(V)^{-1}$ .

Table 1. Factors to polynomial numerical probabilistic models of snow load random process.

Factors of a transformation	Regions of Ukraine according to the ground snow load (Loads 1987)				
function	I	II	III		
$P_{0}$	-0.02662	-0.18393	0.21113		
$P_{1}$	1.18273	0.9563	1.01538		
$P_{2}$	0.15556	0.26648	0.1894		
$P_3$	-0.1361	-0.03891	-0.02464		
$P_{4}$	$-1.91 \cdot 10^{-3}$	$-7.28 \cdot 10^{-3}$	$-2.95 \cdot 10^{-3}$		
$P_{5}$	$7.94 \cdot 10^{-3}$	$1.64 \cdot 10^{-3}$	$2.9 \cdot 10^{-4}$		

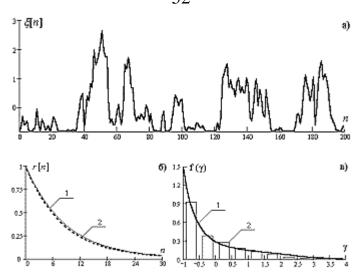


Fig. 1. Model of normalised stochastic process of ground snow load:
(a) realisation of process; (b) NCF of process; (c) density function of ordinates. 1 — idealised values 2 — experimental values.

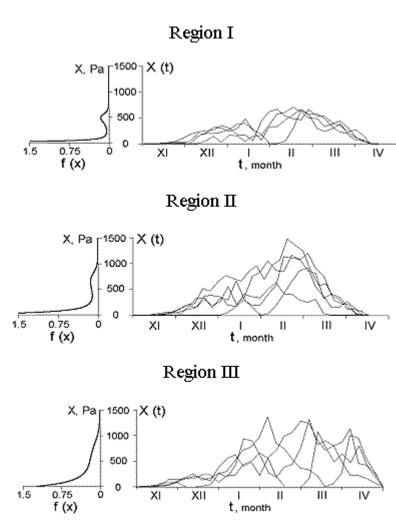


Fig. 2. Annual realisations of ground snow load of quasi-stationary stochastic process for regions of Ukraine.

## 2.4 Definition of numerical model adequacy to real load processes

The parameters of district numerical probabilistic models of the ground snow load for the territory of Ukraine are tabulated in Table 2. The numerical realisations of snow load stochastic processes for different regions of Ukraine were obtained with the help these parameters. The examples of realisations are introduced by Figure 2. Numerical statistical and frequency characteristics are well co-ordinated with experimental data. This adequacy of district numerical probabilistic models to the actual stochastic snow load processes was also tested for quantity of outliers of stochastic process over the given level. Such approach is illustrated by graphs of annual outliers in Figure 3 which it testifies the accuracy of designed models and the possibility of their applications for the solution of reliability problems.

The obtained probabilistic model can be used for revision of parameters of stochastic process of snow load, for solving the problem of simulation of joint action of snow load and other random loads applied to building structures or directly in reliability calculations of structure members under snow and constant loads. Moreover it is possible to receive supplementary data for other forms of presentations of loads on the basis of obtained models in the form of stochastic process.

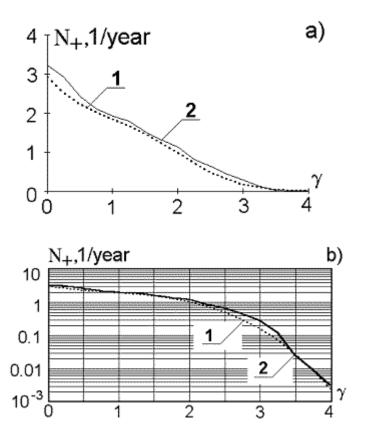


Figure 3. Number of annual outliers of III region of Ukraine:

(a) normal scale (b) logarithmic scale;

1 — experimental data; 2 — by designed model.

## 3 EVALUATION OF STEEL STRUCTURE RELIABILITY UNDER SNOW LOAD

The evaluation of steel structure reliability based on its strength and rigidity in accordance with existing Codes is of great importance for different snow regions of Ukraine (Loads 1987). The considerable part of these structures is made of members of cover structures: roof girders and trusses, purlins. The probability of non-failure member structure for the operation period is taken as a criterion for the structure reliability estimation. A problem of reliability evaluation is divided into two groups depending on the type of borders of permissible conditions area  $\Omega$ 

Table 2. Parameters of district numerical probabilistic models of ground snow load for the territory of Ukraine

	Parameters values for snow					
Types of parameters	regions of Ukraine					
	I	II	III			
Parameter NCF $\alpha$ , 1/24hours	0.0388	0.0241	0.017			
Correlation zone $t_k$ , 24hours	59	95	135			
Discretisation step Δτ, 24 hours	5.0	5.0	5.0			
Length of implementation $T_{win}$ , 24 hours	145	145	150			
Effective frequency $\omega_e$ , 1/ hour	0.0059	0.0039	0.0030			
Frequency on extremes $\omega_m$ , 1/hour	0.0158	0.0120	0.0099			

# 3.1 Definition of steel structure reliability under the random border of area $\Omega - \tilde{\xi}$

A border  $\tilde{\xi}$  is considered to be distributed under the law  $f(\xi)$ . The structure load carrying capacity is determined by the space of normal stresses. The margin function of a load carrying capacity of  $\tilde{Y}(t)$  is presented in the form of the stochastic process

$$\widetilde{Y}(t) = \widetilde{\xi} - \widetilde{u}(t) < 0, \qquad (11)$$

where  $\tilde{u}(t)$  — stochastic process of stresses in a steel member under applying loads.

The characteristics of steel strength and permanent load are random variables. Its changes a described by a normal distribution law. A normalised density function of stochastic process of a margin function of a load carrying capacity  $f_{\gamma}(\bullet)$  will be

received as a difference of normal  $f_{\xi}(\bullet)$  and polynomial-exponential  $f_u(\bullet)$  distributions

$$f_Y(\gamma) = \sqrt{\frac{1+p^2}{2\pi}} \int_{Z_1}^{Z_2} \exp(C_0 + C_1 E + C_2 E^2 + C_3 E^3) \exp(-0.5 Z^2) dZ,$$
 (12)

where 
$$E = -\gamma \sqrt{1 + p^2} + Z p$$
;  $Z1 = D\gamma - 1/(V_u p)$ ;  $D = (\sqrt{1 + p^2})/p$ ;  $p = \hat{\xi}/\hat{u}$ ;  $\gamma = (Y - \overline{\xi} + \overline{u})/\hat{Y}$ ;

 $\bar{\xi}$ ,  $\bar{u}$  and  $\hat{\xi}$ ,  $\hat{u}$  — mathematical expectations and standards of distributions, correspondingly.

Parameter Z2 is determined by the method of selection taking into account the accuracy of the obtained results.

The probability of non-failure of a steel member during time t is determined under the formula

$$P(t) = 1 - Q(t), \tag{13}$$

where Q(t) — failure probability of a member (Pichugin 1995).

$$Q(t) = \frac{\omega_{e(Y)} t}{\beta_{\omega(Y)} \sqrt{2\pi}} f_Y(\beta); \ \beta = \frac{\overline{\xi} - \overline{u}}{\sqrt{\hat{\xi}^2 + \hat{u}^2}},$$
 (14)

where  $\omega_{e(Y)}$  and  $\beta_{\omega(Y)}$  are effective frequency and structure complexity coefficient of stochastic process Y(t). The values  $\beta_{\omega}$  for snow load are given in Table 3.

Snow region of Ukraine	Structure complexity coefficient $\beta_{\omega}$	Characteristic maximum $\gamma_0$ with $t = 50$ years
I	2.677	3.097
II	2.953	3.046
III	3.280	3.197

Table 3. Frequency features of snow load.

The portions of stress in a cover steel member under applying constant  $(D_1)$  and snow  $(D_2)$  loads in common stress are calculated by the formulae:

$$D_{1} = \frac{k \gamma_{f(1)}}{k \gamma_{f(1)} + \mu \gamma_{f(2)}}; D_{2} = \frac{\mu \gamma_{f(1)}}{k \gamma_{f(1)} + \mu \gamma_{f(2)}}$$
(15)

where  $k = q_1^H / s_0$  — ratio of specified value of the uniformly distributed constant load  $q_1^H$  ( $\kappa$ Pa) to the specified value of snow cover weight to 1  $\kappa$  of a horizontal ground surface  $s_0$  ( $\kappa$ Pa);  $\mu$  — a transition factor from the ground snow weight to

the snow load to roof structures;  $\gamma_{f(1)}$  and  $\gamma_{f(2)}$  — a reliability coefficient in terms of constant and snow loads correspondingly.

# 3.2 Determination of steel design reliability under the constant border of area $\Omega$ — $\xi$ = constant

The given group includes the problems of reliability estimation of steel members based on the criterion of deflection. Evaluation of reliability of elements is determined by probability of the non-exceeding a random deflection u(t) values of a permissible deflection  $\xi$  for a time t.

Non-failure probability of steel element is derived by (13), where:

$$Q(t) \approx N_{+}(t) = \frac{K_{win} f_{12}[(f_{u} - \overline{u}_{12})/\hat{u}_{12}]}{\sqrt{1 + (\hat{u}_{1}/\hat{u}_{2})^{2}} f_{1}(\gamma_{0(2)})},$$
(16)

where  $K_{win} = T_{win}/t$  — factor which takes into account a duration of the snow load effect;  $f_u$  — the permissible deflection of a girder is determined according to the Code (Loads 1987);  $\gamma_{0(2)}$  — characteristic maximum of snow load which corresponds to the term of repetition t and is taken from Table 3;  $f_{12}(\bullet)$  — ordinate of the distribution of deflections of constant and snow loads. This distribution is determined by the composition of normalised normal  $f_1(\bullet)$  and polynomial-exponential  $f_2(\bullet)$  distributions:

$$f_{12}(\gamma) = \left[\frac{(1+p^2)}{2\pi}\right]^{\frac{1}{2}} \int_{Z_1}^{Z_2} \exp(C_0 + C_1 E + C_2 E^2 + C_3 E^3) \exp(-0.5Z^2) dZ, \tag{17}$$

where 
$$E = \gamma \sqrt{1 + p^2} - Z p$$
;  $Z1 = -1/V_{u(2)}$ ;  $Z2 = D\gamma + 1/(V_{u(2)}p)$ .

The portions of deflection of a steel cover member under applied constant  $(D_1)$  and snow  $(D_2)$  loads in common deflection are determined by the formulae:

$$D_1 = k/(\mu + k)$$
;  $D_2 = \mu/(\mu + k)$ . (18)

## 3.3 Results of the probabilistic calculations of steel structures under constant and snow loads

The results of calculations are given in Table 4. The realised researches have exhibited a general tendency of increasing the probability of structure failure under reduction of a permanent load and growth of a snow load. It is possible to treat the following steel members as particular dangerous.

a) The steel roof members with the portion of snow load  $C_2 > 0.65$  for the I snow region of Ukraine and  $C_2 > 0.5$  for the II and the III snow regions which do not satisfy the reliability level [P(t)] = 0.95.

b) The steel roof members with the portion of snow load  $C_2 > 0.55$  for the I snow region of Ukraine and  $C_2 > 0.4$  — for the II and the III snow regions which do not satisfy reliability level [P(t)] = 0.99.

The correctness of these considerations is confirmed by the reliability calculation of steel cover girders of various types which are located in the different snow regions of Ukraine. Calculation results are introduced in Table 5.

Table 4. Failure probability of steel roof members under constant and snow loads

	ion of oad	Q (t = 50 years)					
$D_1$	$D_2$	Region I	Region II	Region III			
0.9	0.1	$1.810 \cdot 10^{-5}$	$2.337 \cdot 10^{-5}$	$1.413 \cdot 10^{-5}$			
0.8	0.2	$8.713 \cdot 10^{-5}$	$1.596 \cdot 10^{-4}$	$1.152 \cdot 10^{-4}$			
0.7	0.3	$3.034 \cdot 10^{-4}$	$1.416 \cdot 10^{-3}$	$9.282 \cdot 10^{-4}$			
0.6	0.4	$1.062 \cdot 10^{-3}$	$8.131 \cdot 10^{-3}$	$6.005 \cdot 10^{-3}$			
0.5	0.5	$3.692 \cdot 10^{-3}$	0.0377	0.0283			
0.4	0.6	0.0117	0.1350	0.0966			
0.3	0.7	0.0327	0.3726	0.2468			
0.2	0.8	0.0858	0.8167	0.4997			
0.1	0.9	0.2014	1.0000	0.8473			
0	1.0	0.4214	1.0000	1.0000			

Table 5.Probabilistic calculation of steel cover girders with different roof (span L = 12 m, step B = 6 m) with operation period t = 50 years.

S		Li	mit state of	Limit state of					
/pe	1 group				2 group				
Roof types						Failure pro	obabilities		
Roc	$\mathbf{D}_1$	$\mathbf{D}_2$	Failure probabilities	$D_1$	$D_2$	lower val-	upper val-		
						ue	ue		
			Region	I					
T1	0.86	0.14	1.59·10 <sup>-5</sup>	0.89	0.11	$8.53 \cdot 10^{-3}$	0.416		
T2	0.70	0.30	$1.87 \cdot 10^{-4}$	0.73	0.27	$6.13 \cdot 10^{-5}$	0.611		
T3	0.42	0.58	0.0583	0.52	0.48	$3.30 \cdot 10^{-9}$	2.022*		
			Region	II					
T1	0.82	0.18	$9.29 \cdot 10^{-3}$	0.84	0.16	$8.54 \cdot 10^{-3}$	1.270*		
T2	0.62	0.38	0.0399	0.66	0.34	$6.13 \cdot 10^{-5}$	1.521*		
T3	0.34	0.66	6 0.5491		0.56	$3.26 \cdot 10^{-14}$	2.228*		
			Region 1	III					
T1	0.76	0.24	$3.43 \cdot 10^{-3}$	0.78	0.21	$9.36 \cdot 10^{-9}$	1.038*		
T2	0.54	0.46	0.1177	0.57	0.43	$9.05 \cdot 10^{-11}$	4.963*		
T3	0.26	0.74	0.5984	0.36	0.64	$3.04 \cdot 10^{-16}$	5.184*		

Notes:

- a) Roof types: T1 roof with warmth-keeping layer on concrete panels; T2 roof with warmth-keeping layer on a steel contoured floor; T3 roof without warmth-keeping layer on a steel contoured floor.
- b) D1 and D2 are portions of constant and snow loads, correspondingly.
- c) Failure probabilities under limit state of the 2 group are determined: the lower value under constant load; upper one under constant and snow loads.
- d) Index \* notes the number of outliers in approximate formula (16).

The researches have proved the lack of reliability of steel cover girders under carrying capacity and deformations. In particular it concerns the girders with a light roof. It is illustrated by considerable influence of the random snow load. Moreover there is an unequal degree of reliability of cover structures in different regions of Ukraine. Hence it is necessary to refine the design values of snow load by increasing of reliability coefficients for these structure groups.

### 4 CONCLUSIONS

As a result of these investigations the design numeric probabilistic model of ground snow load and general methodology of steel structures reliability evaluation under constant and snow loads were developed. The obtained reliability evaluation has allowed to detect potentially dangerous types of steel cover structures and to substantiate on the basis of these proposals the improvement of normative calculations.

## **5 REFERENCES**

- [1] Bykov, V. V. 1971. Numerical Simulation in Statistical Radio Engineering. Moscow: Soviet Radio (in Russian).
- [2] Loads and Loadings, 1987. Construction Standards and Rules 2.01.07-85. Moscow: Stroyisdat (in Russian).
- [3] Pichugin, S. F.1995. Reliability Estimation of Steel Elements under Variable Loads. XLI Konf. Naukova KILIW PAN i KN PZITB "Krynica 1995". Krakow-Krynica:Politechnica Krakowska.
- [4] Pichugin, S. F.1998. Probabilistic Models of Snow Load. Technical Meteorology of the Carpathians: TMK-98. Lvov: Okskart (in Russian).
- [5] Pichugin, S.F.2000. Probabilistic Description of Ground Snow Loads for Ukraine. Snow Engineering. Recent Advances and Development. Rotterdam: A.A.Balkema.

**CRANE LOAD ON BUILDING STRUCTURES** 

# PROBABILISTIC DESCRIPTION OF CRANE LOAD ON BUILDING STRUCTURES

### 1 INTRODUCTION

The load which is applied by the bridge cranes is the main for industrial buildings. This very important load generally determines the architecture and design of a building, the size of a structural elements and the choice of materials. Where as the crane load of a random nature needs further investigations.

The goal of this article is to present the numeric and analytical model of the vertical crane load. On its base the analysis of influence of different factors was done and the decrease design coefficients were proposed. This article attacks the problem which is an integral part of the general probabilistic method examined in more details in our previous works [1,2].

## 2 THE ANALYTICAL ANALYSIS OF A VERTICAL CRANE LOAD

The vertical pressure on a bridge crane wheel is determined as follows:

$$\tilde{F} = \left[ \frac{G_b}{2} + \left( \tilde{Q} + G_t \right) \frac{\tilde{a}}{L_C} \right] \frac{1}{n_0} \tag{1}$$

where:  $G_b$  and  $G_t$  are the weights of a bridge and a crane trolley;  $\tilde{Q}$  — a random weight of a carrying load;  $L_c$  — a bridge crane span;  $n_0$  — the number of wheels from the one side of a crane;  $\tilde{a}$  — the distance from a crane wheel to the left crane way if the pressure is determined on the right way (for a left way pressure  $L_c$  —  $\tilde{a}$  is taken).

A random pressure  $\tilde{F}$  is the function of two random arguments  $\tilde{F}$  and  $\tilde{a}$  which are multiplied in the expression (1). In this connection the probabilistic distribution f(F) can be different in accordance with the type of distributions f(Q) and f(a) and their ratio.

Then the distributions of a product Z=XY of random values are studied. "X" is distributed in accordance with a normal law and "Y" is done with an uniform one:

$$f(X) = \frac{1}{\hat{X}\sqrt{2\pi}} \exp\left[-\frac{X^2}{2\hat{X}^2}\right]; -\infty < X < \infty; \overline{X} = 0;$$

$$f(Y) = \frac{1}{\beta - \alpha}; \alpha \le Y \le \beta.$$
(2)

where  $\overline{X}$  and  $\hat{X}$  — mathematics expectation and standard of X variable; a and  $\beta$  — limits of changes of a variable Y.

The distribution function F(z) is derived as [3]:

$$F(z) = P[(X,Y) \subset D] = \iint_{(D)} f(X,Y) dX dY$$
(3)

In the examined case the integral area is the best of  $\beta$  — a width which stretches from  $-\infty$  to the crossing of a hyperbola Z=XY. In this case the crossing section projection on the axis X are of  $Z/\beta$ -Z/a width. Based on these results the formula (3) is derived as follows:

$$F(z) = \int_{-\infty}^{\frac{7}{\beta}} dX \int_{\alpha}^{\beta} \frac{1}{\beta - \alpha} \frac{1}{\hat{X}\sqrt{2\pi}} \exp\left(-\frac{X^2}{2\hat{X}^2}\right) dy + \int_{\frac{7}{\beta}}^{\frac{7}{\beta}} dX \int_{\alpha}^{\frac{7}{\beta}} \frac{1}{\beta - \alpha} \frac{1}{\hat{X}\sqrt{2\pi}} \exp\left(-\frac{X^2}{2\hat{X}^2}\right) dy$$
(4)

These expressions will be used for the derived integral depending on a parameter with variable limits of an integration [4], As a result the distribution density is f(Z)=F'(Z) after the simplification this formula is:

$$f(z) = \frac{1}{\hat{X}\sqrt{2\pi}(\beta - \alpha)} \int_{z/\rho}^{z/\alpha} \frac{1}{X} \exp\left(-\frac{X^2}{2\hat{X}^2}\right) dX$$
 (5)

This expression may be adapted to the vertical pressure (1) then its distribution density will be:

$$f(F) = \frac{L_C n_0}{\sqrt{2\pi} (\beta - \alpha) \hat{F}} \int_b^a \frac{1}{Q} \exp \left[ -\frac{\left(Q - \overline{Q} - G_t\right)^2}{2\overline{Q}^2} \right] dQ$$
 (6)

The limits of integration here are equal:

$$a = \left(F - \frac{G_b}{2n_0}\right) \frac{L_C n_0}{\alpha} - G_t; b = \left(F - \frac{G_b}{2n_0}\right) \frac{L_C n_0}{\beta} - G_t \tag{7}$$

Based on the numeric characteristic theorem of the random values production the following data were obtained [3]:

a) the numeric expectation of a vertical pressure of crane wheel:

$$\overline{F} = \left[ \frac{G_b}{2} + \left( \overline{Q} + G_t \right) \frac{\beta + \alpha}{2L_C} \right] \frac{1}{n_0}$$
(8)

b) the standard of a wheel pressure:

$$F = \left[ \hat{Q}^2 \frac{(\beta - \alpha)^2}{12L_C^2 n_0^2} + \left( \overline{Q} + G_t \right)^2 \frac{(\beta - \alpha)^2}{12L_C^2 n_0^2} + \hat{Q}^2 \frac{(\beta + \alpha)^2}{4L_C^2 n_0^2} \right]^{\frac{1}{2}}$$
(9)

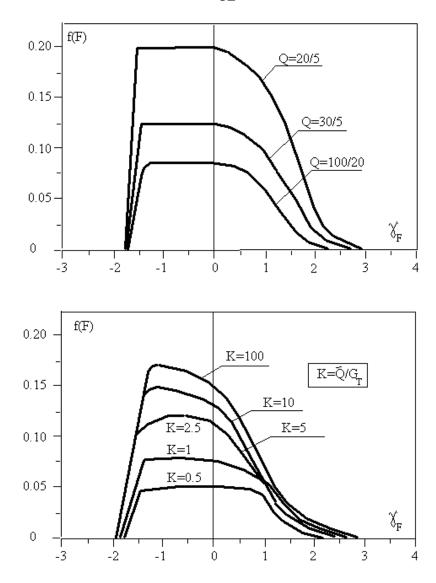


Fig. 1 The influence of different factors on the distribution of vertical pressure of a bridge crane wheel: a) ratio  $K = \overline{Q}/G$ ; b) load carrying capacity.

## 3 THE NUMERIC COMPUTATION OF CRANE LOAD DISTRIBUTIONS

A program for a computation process was done. Its goals were:

- a) ordinate computation of density probability distribution on a formula (6) and the ordinate computation of an integral curve F(F) with the help of a numeric integration;
- b) crane loads numeric characteristics computation: mathematics expectation  $\overline{F}$ , standard  $\hat{F}$ ; coefficient of variation  $V = \hat{F}/\overline{F}$ ; asymmetry A and excess E in accordance with density distribution f(F) calculated on the formula (6);
- c) curve graph of the density probability f(F).

Quite precision of results were obtained when the number of intervals of numeric integration and the distribution ordinates (6) equal 50. The authenticity of obtained results were confirmed by:

a) the coincidence of distributions done by a numeric integration on formula (6) and by a Monte-Carlo method;

- b) the standard condition for a curve (6) should be observed;
- c) the closeness of numeric characteristics  $\overline{F}$  and  $\hat{F}$  derived by formula (8) and (9) and done numerically by a distribution (6).

## 4 THE FACTOR INFLUENCE ON A DISTRIBUTION OF A VERTICAL CRANE LOAD

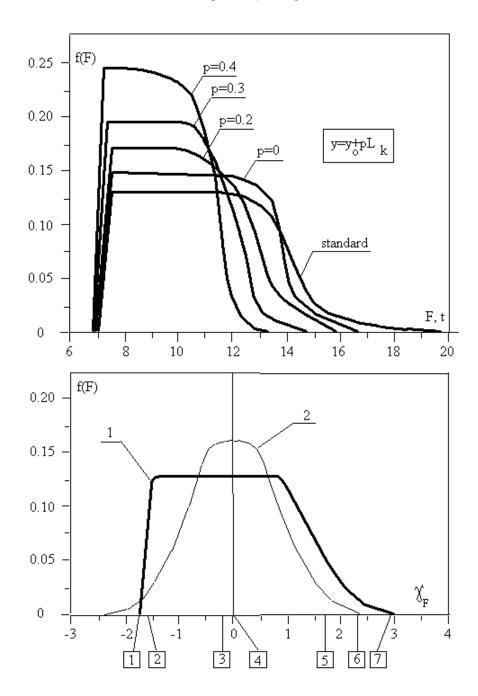


Fig. 2 The influence of a trolley approach (a) and the connection with crane operation (b): 1 — distribution (6), 2 — normal distribution.

Some of vertical pressure wheels distributions of different bridge cranes are shown in fig. 1 and fig. 2. The values of load density distributions f(F) were laid off on the Y-axis and the load values in numeric form (F) were done on X-axis or in a normalized form  $\gamma_F = (F - \overline{F})/\hat{F}$ . For a graph distributions bridge cranes of  $L_c$ =30m span were used. In this case the trolley limit approaches were  $\alpha = y_a = 1.5$  m and

 $\beta = L_c \cdot y_a = 28.5$  m. In accordance with the experimental tests [5] mathematics expectation of a cargo weight was taken as  $\overline{Q} = 0.5Q$  where Q is a crane load-weight carrying capacity. The variation coefficient is  $V = \hat{Q}/\overline{Q} = 0.2$ . The distributions shown in fig. 1 and fig. 2 are not symmetric including its modes shifted to the left from the mathematics expectation, i.e. they're of a positive asymmetry. The distributions are characterized by a fast ascent of a left part, almost a horizontal level of the middle part and smoothly down going of the right branch. As you can see from the fig. 1-a the form of a crane load distribution depends on the ratio of a mean load  $\overline{Q}$  to a trolley load  $G_t K = \overline{Q}/G_t$ . If the trolley load is small and the values of K are large the distributions take the form of a sharp-pointed diagram without a horizontal level. In this case the influence of normally distributed cargo is rather high. The less is K the larger is a trolley load influence and in a final distribution the uniformed one can be more legibly traced. In general the type of received distributions of crane wheel pressure reflects the well-known tendency of random value production distributions to an asymmetric lognormal type.

Bridge cranes of a common type with a flexible hanger have rather narrow parameter interval  $K = 1.15 \div 1.70$ , so their distributions have a very close form as it's illustrated in the fig. 1-b. For the cranes with the rigid hanger with a small K the distributions are specified as closed to the uniformed ones.

As it' depicted in the fig. 2-b the stepped side of a crane load distribution corresponds to the unload trolley (position 1) or to the loaded one with  $Q = \overline{Q}$  (position 2) located at the opposite crane way. The middle part of a distribution is formed the operations with the loaded trolley  $(Q = \overline{Q})$  being located in the middle part of a bridge (position 4) and also with unloaded one at the examined way (position 3, 4). The tag part of a distribution is determined with the trolley operations at the examined crane way with loads, which are in the intervals  $Q = \overline{Q}$  (position 5) and  $Q = \overline{Q} + 2.5\hat{Q}$  (position 6).

The comparison of obtained crane load distributions with the normal ones identifies the similarity of its right downgoing branches as it's illustrated in fig. 2-b. For the typical cranes with a flexible hanger the ordinates of the calculated distributions slightly exceeds the normal ones approximately to  $F \cong \overline{F} + 2.5\hat{F}$ ; if the load level is higher the result is quite opposite. Thus, calculated distributions tags (in accordance with (6)) turn out to be shorter then for the normal distributions with the same numeric characteristics. Based on these results it's possible to use the normal distributions in the examined situation instead of obtained numeric asymmetric curves shown in fig. 1 and fig. 2. The utilization of a normal law for a crane load description was well grounded both theoretically and experimentally [5, 6].

## 5 CONSIDERATION OF A TROLLEY LIMIT APPROACH

It's rather usual situation in situ when the crane loaded trolley doesn't enable to approach quite closely to the crane way because of some barriers as different equipment, buildings etc In this case maneuver possibilities of a trolley are restricted as a result of which the corresponding vertical crane loads distributions are limited as well

without changing general form (fig. 2-a). Specific and design values of crane wheels pressures can be lowed with the help of a decrease coefficient:

$$l = F_n(P)/F_n \tag{10}$$

with  $F_n$  and  $F_n(P)$  — specific wheel pressures with the same provision [P], correspondingly to the standard approach to the examined crane way  $y_0$  and to the limited approach of a trolley  $y_{min}=y_0+p_{Lc}$ .

The analysis has demonstrated that the load level provision [P] doesn't influence greatly the coefficient 1 The results of a 1 coefficient computation in accordance with the formula (10) for cranes with a flexible hanger of different load-carrying capacity in comparison with a stripper-crane with a rigid hanger are tabulated in table 1. The values of the parameter of a limit trolley approach were taken as  $p = 0.1 \div 0.4$ , "1" values are also presented here derived from the recommended formula (11) deduced by the author:

$$l = 1 - \frac{mpL_C}{2L_C + m(L_C - y_0)},$$
(11)

where  $m = (Q + G_t)/G_b$ .

While deducing this formula we consider that the probabilistic character of the wheels pressure with the trolley limited approach and in the case of a normal one will be identical. This supposition leads to some overstating of 1 coefficient in comparison with derived one from the formula (10) and the experimental data.

Rounding up the obtained coefficient 1 the design decreasing coefficients  $l = 0.76 \div 0.94$  with  $p = 0.1 \div 0.4$  were proposed (table 1). These coefficients are offered for the computation of the existing structures in the case of a reconstruction or reinforcing.

### 6 CONCLUSIONS

The analytical expressions for the distribution density and numeric characteristics of a vertical wheel pressure of the bridge cranes were observed. Crane load distributions were derived by the numeric method, graph of crane load distributions was presented. The analysis of such factors as crane carrying load capacity, the ratio of a cargo weight to a trolley, trolley limit approach was done. The decreasing design coefficient  $l = 0.76 \div 0.94$  for a trolley limit approach was proposed. It can be successfully used for checking the restoring and reinforcing structures.

Table 1. Decrease of crane wheel pressure with trolley limit approach

Trolley limit approach	Crane pa	rameters	Coefficients 1					
Р	Q,t	L <sub>c</sub> ,m	Formula (11)	Formula (10)	Recom- mended			
0.1	20/5	22.5	0.965	0.928	0.940			
	30/5	28.5	0.970	0.938				
	50/10	28.5	0.9626	0.9298				
	100/20	22.0	0.9511	0.8688				
	50/20	27.0	0.9553	0.9153				
0.2	20/5	22.5	0.930	0.860	0.880			
	30/5	28.5	0.9398	0.877				
	50/10	28.5	0.9252	0.8626				
	100/20	22.0	0.9023	0.7926				
	50/20	27.0	0.9106	0.8316				
0.3	20/5	22.5	0.8943	0.7910	0.820			
	30/5	28.5	0.9097	0.8170				
	50/10	28.5	0.8574	0.7976				
	100/20	22.0	0.8534	0.7173				
	50/20	27.0	0.8658	0.7482				
0.4	20/5	22.5	0.8575	0.7240	0.760			
	30/5	28.5	0.8797	0.7587				
	50/10	28.5	0.8503	0.7308				
	100/20	22.0	0.8095	0.6418				
	50/20	27.0	0.8211	0.6919				

### 7 REFERENCES

- [1] Pichugin S. Reliability Design Method of Steel Structures. XL Konferencja Naukowa KILiW PAN i Komitetu Nauki PZITB. Rzeszow-Krynica-Warszawa, 1994. (in Russian)
- [2] Pichugin S. Reliability Estimation of Steel Elements Under Variable Loads. XLI Konferencja Naukowa KILiW PAN i Komitetu Nauki PZITB. Krakow-Krynica, 1995
- [3] Ventzel E. The Theory of Probabilities M. Phyzmatgiz, 1962. 564 p.
- [4] G.Korn, T.Korn. Mathematical Handbook for Scientists and Engineers. Mc.Graw-Hile Book Company, New York, Toronto, London, 1961
- [5] Koshutin B. Statistical Determination of Partial Safety Coefficients for Vertical Crane Loads.//Metal Structures, Stroyizdat, Moscow, 1966
- [6] Kikin A., Figarovski A., Pichugin S. The Experimental Data on Cross Horizontal Forces Applied by Different Types of Cranes. // Prom. Stroitelstvo. 1967. Nol2 -p.8-13
- [7] Pichugin S., Levanin Y. The Results of Experimental Research of Vertical Loads of Bridge Cranes on the Columns of the Open-Heart Shops.// The News

of High Educational Establishments. Construction and Architecture, Nol2, 1974. — p.31-35

# ANALYSIS OF BRIDGE CRANE LOADS ON INDUSTRIAL BUILDINGS

### 1 INTRODUCTION

Bridge crane loads are the main ones for industrial buildings. The researches of these loads were done by different Soviet scientific organisations and some high educational institutions 20-30 years ago. Some good results were obtained but the problem of crane loads is still of great interest to the scientists. But lately the studies of this kind were not carried out in the countries of the former Soviet Union. Hence the development of reliability computation and new Building Codes requires authentic and probabilistic evaluation of bridge crane loads. For many years the author has been performing the work which deals with the crane loads [1-4]. The data mainly reported in this paper are experimental ones and present the loads of different types of cranes. These data were obtained in difficult and complicated experiments carried out in the shops of industrial plants.

## 2 METODOLOGY OF EXPERIMENTAL RESEARCHES OF CRANE LOADS

Continuous crane loads registration for 3-18 days and nights was performed. Dynamometers of a slot-type were used for measuring the shear forces. They were fixed in the place of upper belts attachment of crane beams to the columns. All horizontal loads of working bridge cranes were transmitted to these dynamometers. Two sets of measuring instruments were used:

- the inductive measuring detectors which were recording the deformation of dynamometers, 10-channel rectifying and filtering block and magnet and electric oscillograph;
- mechanical recorder of a continuous action.

For measuring of vertical crane loads the inductive detectors were included into strain gauges with the base of 500÷1000 mm. These strain gauges were placed to the supporting parts of crane beams or to the lower part of the columns. The visual methodology of vertical crane loads study was applied [5].

### 3 THE OBJECTS OF RESEARCH

The crane load studies were performed at three Russian metallurgical plants in different shops from 10 to 30 years of service. Vertical and horizontal loads of powerful casting crane were under investigation in Martin Shops (table 1 and N 7-10 in table 2).

The loads of cranes with flexible hanging cargo were analysed:

- in stripping cranes (pos. 1. table 2);
- in slab stores and in stores of finished articles of 4 rolling shop with pratsen cranes (pos. 2 and 3);
- in heating soaker pits of slabbing and blooming equipped with ingot tongs cranes;

- in a furnace span of Martin shop equipped with charging cranes (pos. 6). The loads of cranes with flexible hang cargo were examined:
- in testing shop with a cracker and on the scrap with magnet and trestle equipped cranes (pos. 11 and 12);
- in the place where mould is limed and the cranes are of general type (pos. 13);
- in the area of magnet materials equipped with magnet cranes (pos. 14);
- in the area of materials with grab cranes (pos. 15).

## 4 DISTINCTION FEATURES OF CRANE LOADS

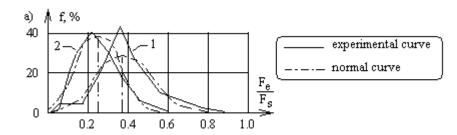
The experimental data were statistically analysed, the obtained results are tabulated in tables. In accordance with this the following three features of crane loads with random character were determined.

- d) Comparatively fat identification and stabilisation of probabilistic distributions of crane loads with the account of comparatively small number of measures. The further increase of statistic amount of information doesn't change the results either (in degree) quantitatively or qualitatively (in kind). This is just both for crane numeric characteristics and parameters of frequency. Taking into account the mentioned above facts the period of statistic experiment can be comparatively short for metallurgical cranes that work extremely intensive. So the crane load is stationary and ergodic.
- e) Good correspondence of distributions of vertical and horizontal crane loads to normal law. That can be clearly seen at fig.1 and illustrated by the data in 1 and 2 tables and by our results [1, 2] and by the researches of other authors [5].
- f) Majority of crane loads which mainly form their statistic distributions (fig. 1) are the loads of one bridge crane with small loaded trolley. at the same time the loads of two and more cranes with cargo close to the load carrying capacity are very seldom.

## 5 THE RESULTS OF RESEARCHING VERTICAL CRANE LOADS

The samples of obtained experimental distributions of vertical loads are presented at fig. 1-a. The relative quantities were laid off on the abscissa axis and they were equal to the ratio of real loads Fe to specified crane load Fs. The later was determined by two closely stand cranes in every span. The relative mean values of  $\bar{F}$  and the standard  $\hat{F}$  of the load (table 1) were calculated in much the same way.

The obtained experimental distributions of the vertical crane load for the middle rows of columns are shifted to the starting point of co-ordinates in comparison with the distribution for the edge rows (fig 1-a). In demonstrates that in that case the structures of middle rows are substantially under loaded in comparison with the edge rows. It's connected with the specific technological process of Martin shops. The problem is that the most of bridge cranes in Martin shops work with under load trolleys placed in the edge rows or in the middle of shop spans.



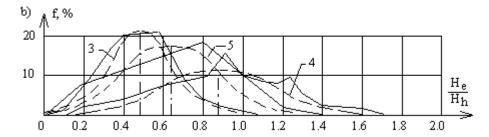


Fig. 1. Distributions of Crane Loads

a) vertical loads of Martin-Shop: 1 — edge row; 2 — middle row;
b) horizontal loads: 3 — Martin-Shop with normal gauge; 4 — Martin-Shop, area with broaden gauge; 5 — Stripping Shop.

In that connection it's proposed to establish the coefficient of the load of column rows of vertical crane load. It's determined as

$$p = \gamma_f^e / \gamma_f \,, \tag{1}$$

where  $\gamma_e$  — the load coefficient determined to the crane load with a cycle round of 20 years;  $\gamma_f$  = 1.1 — the load coefficient for crane loads in accordance with Building Code [6].

The coefficients p for Martin shops are close to "1" for the edge rows and are much more lower (0.50-0.73) for the middle rows. The additional reserves are provided for middle row columns if (when) crane loads for both spans ( $\gamma_f^e = 0.495-0.62$  in table 1) are taken into account. If the movement of the loaded trolley is restricted it's possible to use our corrective coefficients [4].

Table 1. Results of experimental researches of vertical crane loads

		Mart	in shop — 1							
	Castir	ng aisle	Furnace	Casting						Methodology with
			aisle	and fur-		Visu		the use of apparatus		
				nace						
Parame-				aisles	Casting	g aisle	Casting aisle			
ters								furnace		
					ais					
	Row	Row	Row	Row mid-	Row edge	Row	Row	Row	Row middle	Row edge A
	edge	middle	middle III	dle III	A	middle	edge D	middle	В	
	II	III				В		В		
$\overline{X}$	0.373	0.271	0.200	0.162	0.340	0.185	0.286	0.324	0.234	0.500
$\hat{X}$	0.136	0.100	0.068	0.083	0.126	0.043	0.126	0.083	0.044	0.110
S	0.083	0.467	0.694		0.011	0.066	0.358	1.228		0.308
Е	0.746	0.568	0.730		3.182	0.668	0	2.632		1.815
$\gamma_f^e$	1.05	0.77	0.57	0.62	1.06	0.55	1.00	0.80	0.495	0.99
p	0.95	0.70	0.52		0.96	0.50	0.50 0.91 0.73			0.90
n	8	5.0	35.0		8.1	10.0	1.	3.8		11.4

Symbols:  $\overline{X}$  — statistic means value;  $\hat{X}$  — standard deviation; S — asymmetry; E — excess;  $\gamma_f^e$  — experimental load factor of vertical crane load; p — coefficient of column row load; n — number of loads on columns per hour.

## 6 THE RESULTS OF HORIZONTAL CRANE LOAD RESEARCH

As it is widely known horizontal cross loads of bridge cranes are of complicated physical nature and consist of two variables:

- some braking force which appears as the result of cranes trolleys brake; this component is included into Building Codes [6];
- shear forces which arise under the crane bridge movement on the rail-riding track.

The experimental researches [1, 5] demonstrate that the brake forces are not strong in comparison with the shear ones and for many cranes can be ignored. So the horizontal cross loads estimation of bridge cross can be treated like shear forces estimation. These forces can be considered like friction ones and they occur as the result of cross crane wheels shear on the crane rails.

The experimental numeric characteristics of statistic shear forces distributions of cranes of different functions are introduced in table 2. The symbols are follows in this table: Q — crane hoisting capacity; K — the number of crane wheels;  $L_k$  — crane span; B — the base of crane; V — the coefficient of variation. The rest symbols are presented in table 1. The table data are obtained by the author except to the crane  $N_2$  8-10 which were searched by B.N.Koshutin [5].

The experimental distribution samples are depicted in fig 1-8. Taking into account the mentioned above fact about crane loads as the main parameter of shear force estimations the characteristic shear was introduced and defined as

$$H_h = 0.1 \cdot F_{mid}^w \cdot \sum Y, \tag{2}$$

where  $F_{mid}^{w}$  — vertical pressure of a crane wheel with unloaded trolley situated in the middle of a bridge;  $\sum Y$  — the sum of ordinates of the influence line for columns or crane beams etc. taking account of the 1st crane; 0.1 — the ratio coefficient.

The relevant values were laid off on the abscissa (fig 1-b) which equals the ratio of existing shear forces  $H_e$  to the values  $H_h$  in accordance with (2) the relevant values  $\overline{H}$  and H were determined by the same way which is shown in table 2. In accordance with the experimental results the following shear force groups were specified (table 2):

- the 1st group the crane loads with rigid hang cargo and heavy trolleys which arise great dynamic affect. For these types of cranes the shear force mean values are  $H_e = (0.55 \div 1.0) H_h$ , the largest values are
- the coefficient of variation V=0.5 and the ratio  $H_{rec} = H_h$  can be proposed for reliability evaluation with some reserve;
- the 2nd group the loads of heavy multi-wheeled cranes with flexible hang cargo and large loading capacity. Two groups of distributions are introduced for these cranes:
  - g) the mean values are  $\overline{H}_b = (0.5 \div 0.7) H_h$  for the areas with normal gauge for crane rails the largest ones  $H_{\text{max}} = (1.1 \div 1.7) H_h$ ; it's recommended to use  $\overline{H}_{rec} = 0.8 H_{h1}, V = 0.45$ ;

Table 2. Experimental numeric parameters of distribution of crane horizontal shear forces

No. of group	Group of cranes		No. of cranes	Cra	ane pa	rameters		Nun	neric p	arame	ters of s	hear
				<i>Q</i> , t	K	$L_k$ , m	<i>B</i> , m	$\overline{H}$	$\hat{H}$	V	S	E
			1	75/25	16	27.0	9.3	0.80	0.35	0.44	-0.15	-0.71
			2	15	8	28.0	9.5	0.84	0.46	0.55	0.30	-0.60
τ	Cranes	with	3	18	4	28.0	7.8	0.55	0.34	0.62	0.25	-0.94
I	the rigid han	iged cargo	4	10/10	8	24.5	4.9	1.00	0.47	0.47		
			5	30/50	12	32.0	9.4	0.67	0.38	0.57		
			6	5/15	4	19.4	5.8	0.99	0.32	0.33	-0.20	-0.75
	Multi-wheeled Areas with a		7	175/50	16	23.3	7.5	0.47	0.21	0.45	0.30	-0.39
	cranes with	normal	8	350/75	16	22.0	7.5	0.74	0.33	0.45	-0.08	-0.06
	flexible	gauge										
II	hanged cargo	Areas with	9	175/50	16	23.3	7.5	1.15	0.41	0.36	-0.09	-0.21
		narrow and										
		broaden	10	125/30	16	25.5	7.5	1.04	0.38	0.37	-0.57	-0.37
		gauge				• • • •		101	0.10		0.4	0.00
			11	15+15		28.0	5.9	1.06	0.40	0.38	-0.21	-0.20
	4 1 1 . 1 .		12	10/10		32.0	4.6	2.45	0.86	0.35		
III	4 wheeled co		13	30/5	4	22.5	5.3	0.73	0.39	0.54		
	flexible hanged cargo		14	15		28.5	4.4	1.75	0.63	0.36		
			15	23		22.0	4.4	2.80	1.00	0.36		

- h) for the narrow and broaden gauges which exceed 40 mm, where  $\overline{H}_e = (1.0 \div 1.2) H_{h1}$ ,  $\overline{H}_{max} = 2.1 H_h$ ,  $\overline{H}_{rek} = 1.2 H_h$ , V = 0.36;
- the 3rd group the loads of relatively light four wheeled cranes with flexible hang cargo having large span ratio  $L_k$  to the B base and as a result can cause the cocking of a bridge when it moves. These loads significantly exceed the loads of the previous groups and are: mean value of  $\overline{H}_e$  to  $2.8H_h$ , maximum to  $5H_h$ , the average coefficient of variation is V=0.36.

## 7 CONCLUSIONS

The methodology for crane loads registration with the usage of some equipment has been developed and put into practice in the working shops. The experimental statistic parameters of vertical and horizontal loads of bridge cranes of different types were obtained. These parameters can be used for practical evaluation of structure reliability. The significant difference of loading of a vertical crane load of the structures of different types working in Martin shops was exposed. Shear crane loads are divided into 3 groups: for cranes with rigid hang cargo, for multi-wheeled and four-wheeled cranes with flexible hong cargo.

#### 8 REFERENCES

- [1] Kikin A. I., Figarovsky A. V., Pichugin S. F.: Experimental Data on Horizontal Shear Forces Induced by Casting Cranes. Prom. Stroitelstvo, Moscow (1967), N12, p. 8-13.
- [2] Pichugin S., Levanin Y.: The Results of Experimental Research of Vertical Loads of Bridge Cranes on the Columns of the Open-Heart Shops, the News of High Educational Establishments Construction and Architecture (1974), N12, p. 31-35.
- [3] Pitchouguine S.: Certaines particularites de l'analyse stochastique des surcharges sur les constructions des batiments, Problemes de la construction en climat tropical, Conakry (1986), p. 85-94.
- [4] Pichugin S.: Probabilistic Description of Crane Loads on Building Structures, XLIII Konferencja Naukowa KILIW PAN I KN PZITB, Poznan-Krynica (1997), t.3, p. 171-178.
- [5] Kikin A. I., Vasiliev A. A., Koshutin B. N., Uvarov B. Y., Volberg Y. L.: Increase of Metal Structures Durability for Industrial Buildings, Moscow (1984).

## STOCHASTIC PRESENTATION OF BRIDGE CRANE LOADS

### 1 INTRODUCTION

This work continues the number of author's publications devoted to the investigation of the crane loads [1, 2]. These loads are very important as they determine the structural estimation and reliability of the industrial buildings. The main features of crane loads were determined in our previous work [2]: crane load is stationary and ergodic; distributions of vertical and horizontal crane loads are well corresponded to normal law; majority of crane loads are the loads of one crane with small loaded trol-

ley. This article develops the probabilistic presentation of the crane load on the base of different stochastic load models.

## 2 PROBABILISTIC MODELS OF CRANE LOAD

The systematic analysis of five most commonly used probabilistic presentation of random crane load with the account of unspecified and normal laws of distribution are introduced in this work. For these models the solutions of the direct problem of the calculation of the crane load level  $\gamma(t)$  corresponding to the given probability Q(t) and opposite problem of the determination of Q(t) of exceeding  $\gamma$ -level during t were obtained. The normalised load level was taken into account  $\gamma = (x - \bar{x})/x$ , where x — the load ordinate,  $\bar{x}$  — mathematical expectation, x — standard deviation. The evident advantage of all these solutions is the obvious account of a time factor "t".

The main probabilistic model of crane load is the model presented in the form of a stationary random process. The number of outliers of this process  $N_+(\gamma,t)$  gives the estimation Q(t) (opposite problem)

$$Q(t) \cong N_{+}(\gamma, t) = \omega t f(\gamma) / \sqrt{2\pi} = \omega t \exp(-\gamma^{2}/2) / \sqrt{2\pi}, \qquad (1)$$

where  $\omega$ — effective frequency of crane load random process,  $f(\gamma)$ -density of distribution of random process ordinate.

The direct problem in this case can be solved in the following form

$$\gamma(t) = (2Ln\{\omega t/[2\pi Q(t)]\})^{1/2}.$$
 (2)

The more complicated probabilistic problems are solved in the manner of a random process which, however, are difficult for description and take much more computation time. More accessible and simple models mentioned further are based on the random values and the corresponding frequent characteristics and provide not less exact solution if they are proper grounded.

The model of the absolute maxima of random process treats the largest crane loads, which are higher then characteristic maximum level  $\gamma_o$  [3]. This level is a solution of the following equation  $N_+(\gamma_o; 0 \le \tau \le t) = 1$ . After simplification opposite and direct problems for this model were resolved as

$$Q(t) = f(\gamma)/f(\gamma_o) = \exp[(\gamma_o^2 - \gamma^2)/2], \tag{3}$$

$$\gamma(t) = \left[\gamma_o^2 - 2Ln Q(t)\right]^{1/2}. \tag{4}$$

The model in the form of a random sequence of independent crane loads is very often introduced. In this case the frequent parameter is the load intensity  $\lambda$  which equals the number of a load in per-unit time  $t_{\lambda}$ . Possibility of overloading of  $\gamma$  (opposite problem) and corresponding normalised level (direct problem) were determined in the form of

$$Q(t) = \lambda t \left[ 1 - F(\gamma) \right] = \lambda t \left[ \int_{\gamma}^{\infty} \exp(-\gamma^2/2) d\gamma \right] / \sqrt{2\pi} , \qquad (5)$$

$$\gamma(t) = \left(2Ln\left\{\lambda t/\left[Q(t)\sqrt{2\pi} \mu(\gamma)\right]\right\}\right)^{1/2},\tag{6}$$

where  $F(\gamma)$  — integral distribution function,  $\mu(\gamma) = f(\gamma)/Q(\gamma)$  — distribution intensity.

The probabilistic model of crane load in the form of discrete presentation uses the time parameter — mean duration of overloading  $\overline{\Delta}$  connected with the intensity by the ratio  $\overline{\Delta} = t_{\lambda}/\lambda$ . In that connection the calculated form of this model can be represented by the substitution to the equations (5), (6) of the proportion  $\lambda = t_{\lambda}/\overline{\Delta}$ .

The analysis of the problem has demonstrated that the crane load value sampling, which described by normal law can be classified as the exponential type [4]. That's why their maximum values can be presented correctly by double exponential Gumball distribution of a normalised type

$$y = \alpha_n(\gamma - u_n); \ \gamma(t) = u_n + y/\alpha_n \ , \tag{7}$$

where  $u_n$  — characteristic extreme;  $\alpha_n$  — extreme intensity; y = -Ln[-Ln F(t)] — argument of the Gumball distribution.

These extreme parameters are connected with the volume of body sampling ne as [4]

$$n_e^{-1} = 1 - F(u_n), \alpha_n = n_e f(u_n),$$
 (8)

where F and f— integral and differential functions of the initial normal distribution.

With the usage of the normal law the formula (7) was transformed

$$y(t) = \mu(u_n) \left[ \gamma(t) - \left( 2Ln \left\{ \left[ n_e / \left[ \mu(u_n) \sqrt{2\pi} \right] \right] \right\} \right)^{\frac{1}{2}} \right]. \tag{9}$$

From expressions (8) the corresponding volume  $n_e$  was obtained

$$n_e = \mu(u_n)\sqrt{2\pi} \exp(-u_n^2/2)$$
 (10)

## 3 MEAN LOAD PARAMETERS FOR DIFFERENT CRANES

The experimental crane probabilistic parameters of all examined models are tabulated in table 1. They are classified in the function of the hang cargo type (flexible or rigid) and crane behaviour (4K-8K). The values  $\bar{x}$  and  $\hat{x}$  which fully determine the normal distribution of crane load are tabulated there. In the table 1 the corresponding frequent characteristics are shown. The numeric values of parameters of different models were derived from the condition of close evaluation Q(t) for the service life t=50 years.

The worked out method which examined the models and obtained parameters gives possibility to estimate reliability of steel structures under crane and other loading [5].

### 4 RELIABILITY OF STEEL ELEMENTS UNDER CRANE LOAD

Crane beams as well as crane trestle structures can be treated like these elements. We'll examine only general stress of elements without local effects and fatigue. In this case the probability of steel structure failure Q(t) depends on the ratio of a load (strength) of one bridge crane  $X_{M1}$  to the load of two cranes  $X_{M2}$  then on the ratio of a load — carrying capacity of crane to its mass and also on crane behaviour. The analysis showed that steel elements have a deficient reliability if they are under four-wheeled crane loads when  $X_{M1}/X_{M2} \ge 0.8$  i.e. in the case of one crane force has dominance. In the rest cases steel elements reliability determined on the base of general stress state criteria under crane loads is sufficient.

In some cases the crane load can be corrected when the structural reliability is preserved. For example in the case if the approach of a loaded crane trolley to a crane way is limited the crane load can be decreased with the help of lower coefficient l = 0.76 - 0.94 [1].

Table 1. Probabilistic parameters of crane load

No. of models	Madal managatan	Cymb ol	Unit	Numeric values for different hang cargo and crane behaviour			
	Model, parameter	Symbol	Unit	flexible			rigid
				4K-6K	7K	8K	8K
	Common						
	parameters						
	Mathematical expectation	$\overline{x}$		0.66-0.243η		0.758	
	Standard deviation	$\hat{x}$		0.131			0.274
1	Normal stationary						
	random process						
	Effective	ω	1	71.0	106.8	215	215
	frequency	W	24hours	71.0	100.8	213	213
2	Absolute maxima						
	of random process						
	Characteristic	$\gamma_0$		4.796	4.881	5.022	
	maximum level						
3	Random sequence		_1		06 246406		6
	Load intensity	λ	year <sup>-1</sup>	$1.42 \cdot 10^6$	$2.16 \cdot 10^6$	$4.44 \cdot 10^6$	
4	Discrete						
	presentation						
	Mean duration of overloading	$\overline{\Delta}$	second	22.2	14.6	7	<b>7.1</b>

5	Normal extremes				
	(per year)		_	_	_
	Body sampling	$n_e$	 $1.26 \cdot 10^6$	$2.09 \cdot 10^6$	$3.49 \cdot 10^6$
	Characteristic ex-	11	4.8	4.9	5.0
	treme	$u_n$	 4.0	4.2	3.0
	Extreme intensity	$\alpha_{n}$	 4.99	5.09	5.19

Notes.

- a) Values of  $\bar{x}$  and  $\hat{x}$  are the relative quantities equal to the ratio of real load parameters to specified load of one crane.
- b)  $\eta = Q/G_c$  crane load characteristic, Q load-carrying capacity,  $G_c$  crane mass.

Table 2. Temporary factor  $\gamma_T$  for computation of a structure under crane load

Structure service	1 month		Years	
term T	1 month	1	10	50
$\gamma_{\scriptscriptstyle T}$	0.88	0.93	0.975	1.0

The analysis of steel element reliability with the limited service term was performed. It demonstrated that the design crane load for these elements could be decreased by temporary factor  $\gamma_T$  multiplication in accordance with table 2.

In accordance with the Building Codes of formed USSR [6] steel beams for crane with intensive behaviour have to be designed for shear force effect of bridge cranes. The recommendations with regards to these effects are given in our article [2]. It's also recommended to introduce the coefficient of load combination  $\psi$ =0.9 to the steel beam design if both shear and vertical crane loads are taken into account. This recommendation is just for strength ratio  $X_{M1}/X_{M2} < 0.8$  if the part of every load (vertical of shear) in joint strength is not less then 30%.

## 5 CONCLUSIONS

The large amounts of statistic results on crane load were examined for the bridge cranes of the different types. The most widely spread probabilistic models of crane load were observed. They are as follows: stationary random process and its absolute maxima, random sequence of independent loads, discrete presentation and extreme model. Having integrated some initial date all necessary mean parameters of mentioned probabilistic crane models were determined. The worked out method and parameters are given possibility to estimate the reliability of crane steel structures and to propose new design coefficients.

## **6 REFERENCES**

[1] Pichugin S., Probabilistic Description of Crane Loads on Building Structures. XLIII Konferencja Naukowa KILIW PAN i KN PZITB. Poznan-Krynica, 1997, t.3, p.171-178.

- [2] Pichugin S., Analysis of Bridge Crane Loads on Industrial Buildings. XLIV Konferencja Naukowa KILIW PAN i KN PZITB . Poznan-Krynica, 1998, t.7, p.41-47.
- [3] Bolotin V., Statistical Methods in Structural Mechanics. San Francisco, Holden-Day, 1969.
- [4] Gumbel E., Statistics of Extremes. New York, Columbia University Press, 1967.
- [5] Pichugin S., Reliability Estimation of Steel Elements under Variables Loads. XLI Konferencja Naukowa KILIW PAN i KN PZITB. Krakow-Krynica, 1995, p.151-156.
- [6] Loads and Loadings. Construction Standards and Rules 2.01.07-85. Moscow, 1987.

## STOCHASTIC PRESENTATION OF BRIDGE CRANE LOADS

## **SUMMARY**

The large amounts of statistic results on crane load were examined for the bridge cranes of the different types. The most widely spread probabilistic models of crane load were observed. They are as follows: stationary random process and its absolute maxima, random sequence of independent loads, discrete presentation and extreme model. Having integrated some initial date all necessary mean parameters of mentioned probabilistic crane models were determined. The worked out method and parameters are given possibility to estimate the reliability of crane steel structures and to propose new design coefficients.

RELIABILITY OF ELEMENTS AND STRUCTURES

## RELIABILITY OF STEEL TUBULAR TRUSSES

#### **SUMMARY**

The steel tubular truss roofs are widely used in an industrial construction. The distinction features of trusses are their bending compressive and bending tensile carrying load capacity under the dead and snow load. The components of dead load are presented like random values distributed by Gauss law. The snow load is described by probabilistic model of quasi-stationary differentiated random process with polynomial-exponential distribution of an ordinate. The function of stability reserve is offered. The possibility of its linearization is suggested. The numeric experiment for the estimation of reliability parameters of examined 18, 24, 30 m span trusses has been done. The derived data demonstrate the existence of truss elements with considerably different degrees of reliability as well as the deficient reliability of some web bars. The recommendations for the truss design correction are given.

KEYWORDS: Steel truss, tubular structure, reliability, probabilistic model.

## 1 STEEL TUBULAR TRUSS ROOFS

The steel tubular truss roofs are widely used in an industrial construction not only in Ukraine but in CIS countries as well. The roofs include roof trusses of 18, 24, 30 m span with thin-walled right-angled bend-welded steel tubes (fig. 1). The distance of trusses equals 4m. The roof trusses are placed on under-roof trusses of 12m span which are supported by the steel short columns. The galvanised profiled roof steel with thermal insulation and rubberoid roofing are fixed on the truss top chord. The truss lower chords are strengthened by vertical bracing and cross bars. The roof trusses and under-roof ones are hinged. The top and lower chords and two diagonals near support are manufactured using alloy steel 09Γ2C, other web bars are made from carbon steel BCτ3κπ6. The distinction features of trusses like these are their bending compressive and bending tensile carrying load capacity under the dead and snow loads.

## 2 TOPICS OF PROBABILISTIC STRUCTURE ANALYSIS

The reliability parameters estimation of beam-column elements is based on the general procedure proposed by V. V. Bolotin [1]. The stochastic behaviour of the structure may be described as random walking v(t) in the space of the structure performance quality v (fig. 2). The structural element fails when the process v(t) exceeds the closed subspace  $\Omega$  of permissible structure states. The structure failure problems connect with random jumps of a stochastic process v(t) over a given boundary of the subspace  $\Omega$ .

In our case the failure condition corresponds to an exceeding of the stable equilibrium boundary:

$$\widetilde{Y}(t) \ge \widetilde{\sigma}_{cr}(t) - \widetilde{\sigma}_{N}(t),$$
 (1)

where  $\tilde{Y}(t)$  — reserve stability function;  $\tilde{\sigma}_{cr}(t)$ ,  $\tilde{\sigma}_{N}(t)$  — random processes of critical and axial stresses, correspondingly.

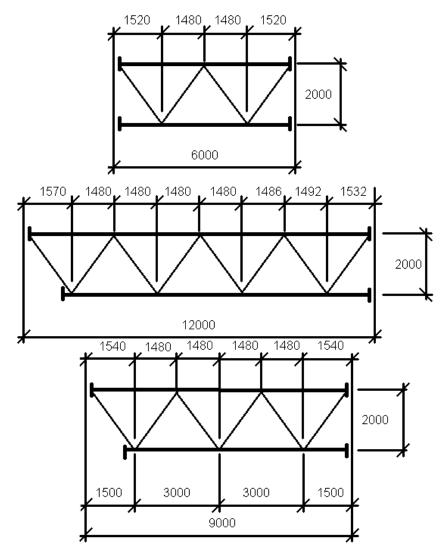


Fig. 1. Schemes of steel tubular roof structures.

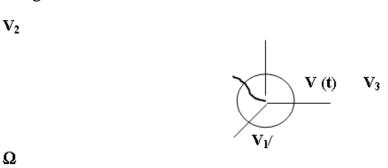


Fig. 2. Illustration of system stochastic behaviour.

## 3 PROBABILISTIC MODELS OF LOADS AND STEEL RESISTANCE

Ukrainian winters are changeable with little to much snow. During winter, snow load has two little transitional irregular parts. The beginning of winter is the period of snow accumulation, the end of winter is the snow melting stage. For the main

winter period it is possible to use bimodal polynomial-exponential distribution with the follows normalised presentation [2]:

$$f(\gamma) = \exp(c_0 + c_1 \gamma + c_2 \gamma^2 + c_3 \gamma^3), \tag{2}$$

where  $\gamma = (x - \bar{x})/\hat{x}$  — normalised deviation of snow load;  $\bar{x}$  and  $\hat{x}$  — snow mathematical expectation and standard deviation, correspondingly;  $c_0 - c_3$  — polynomial coefficients of the exponent argument.

The components of dead load are presented like random values distributed by Gauss law. The dead load is characterised by little coefficient of variation  $V_D$ =0.1-0.2. The probabilistic model of the snow load is presented in the manner of a random quasi-stationary process (PR) both with the constant district coefficients of variation and frequency parameters. A steel strength was determined using the standard experimental method. This steel characteristic is variable; it is presented like normal random value.

If loads are presented in the form of random processes and the time factor is taken into account the reserve stability function (1) is the random process as well. In this case the failure of an element takes place when a stochastic load stress  $\tilde{S}(t)$  exceeds the random resistance of the element  $\tilde{R}_y$  (fig. 3). The loads on the existing structures are of repeated character. As a result the ideal Prandtle diagram can be used in the reliability analysis of the steel elements. Proposed method takes into account the initial imperfections of structural elements including bar flexures, eccentric applications of loads etc [3].

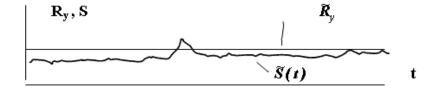


Fig. 3. Graphic interpretation of structure failure.

## 4 RELIABILITY ESTIMATION OF STEEL TUBULAR TRUSS ROOFS

Non-nodal load charges the tubular trusses outside their nodes. As a result the reliability analysis of truss elements has some specific distinctions. The random longitudinal force  $\tilde{N}$  and bending moment  $\tilde{M}$  were determined as the functions of a dead load  $\tilde{q}_d$  and snow load  $\tilde{p}_s$  by means of the coefficients  $\alpha_d$ ,  $\beta_d$  and  $\alpha_s$ ,  $\beta_s$ . Therefore the efforts derived from the following formulae:

$$\widetilde{N} = \alpha_d \widetilde{q}_d + \alpha_s \widetilde{p}_s, \tag{3}$$

$$\widetilde{M} = \beta_d \widetilde{q}_d + \beta_s \widetilde{p}_s. \tag{4}$$

The functions (3) and (4) are interdependent, their dependence is characterised by the correlation moment  $K_{NM}$  and correlation coefficient  $r_{NM}$ . These parameters can be determined as follows:

$$K_{NM} = \alpha_d \beta_d \hat{q}_d^2 + \alpha_s \beta_s \hat{p}_s^2, \tag{5}$$

$$r_{NM} = \frac{K_{NM}}{\hat{N} \cdot \hat{M}},\tag{6}$$

where  $\hat{q}_d$ ,  $\hat{q}_s$ ,  $\hat{N}$ ,  $\hat{M}$  — standard deviation of dead and snow loads, longitudinal force and bending moment, correspondingly.

As mentioned in the previous work [4], it is possible to apply linearization procedure for the reserve stability function  $\tilde{Y}$ :

$$\widetilde{Y} \cong \varphi(\overline{\sigma}_{v}, \overline{M}, \overline{N}) + j_{1}\widetilde{\sigma}_{v} + j_{2}\widetilde{M} + j_{3}\widetilde{N},$$
 (7)

where  $j_1$ — $j_3$  — function  $\tilde{Y}$  partial derivatives of the stochastic arguments. The transformation of the formula (7) taking into account the expressions (3) and (4) allow to exclude the correlative connections of arguments and to obtain an equation:

$$\widetilde{Y} = j_1 \widetilde{\sigma}_{v} + \widetilde{q}_d A_d + \widetilde{p}_c A_s, \tag{8}$$

where  $A_d = j_3 \alpha_d + j_2 \beta_d$  and  $A_s = j_3 \alpha_s + j_2 \beta_s$  — the transitional coefficients for a dead and snow load.

The normalised distribution density of stability reserve function was determined with the help of numerical integration. This density was calculated as the difference of normal law (steel strength, dead load) and polynomial—exponential distribution (snow load) (fig. 4):

$$f(\beta_Y) = \sqrt{\frac{(1+p_1^2)}{2 \cdot \pi}} \cdot \int_{Z_1}^{Z_2} e^{C_0 + C_1 \cdot E + C_2 \cdot E^2 + C_3 \cdot E^3} \cdot e^{-0.5 \cdot Z^2} dz, \tag{9}$$

where 
$$E = -Z_Y \sqrt{1 + p_1^2} + Z \cdot p_1$$
,  $D = \frac{\sqrt{1 + p_1^2}}{p_1}$ ,  $Z_1 = D \cdot \beta_Y - \frac{1}{v_c \cdot p_1}$ ,  $Z_2 = 5 \div 10$  — upper

limit of integration which is determined with the account of necessary calculation accuracy for  $f(\beta_Y)$ ;  $p_1 = \frac{\hat{x}_N}{\hat{x}_c}$ —ratio of normal and polynomial-exponential standards;

 $\beta_Y = (Y - \overline{Y})/\hat{Y}$  — standardised variable of stability reserve;  $v_c$  — variation coefficient of snow load.

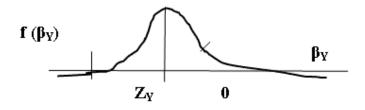


Fig. 4. Normalised distribution of stability reserve.

The initial probability of the structure failure was determined with the help of numerical integration of the distribution density  $f(\beta_Y)$  with the lower limit of integration  $-\infty$  and upper one  $Z_Y$ . The safety characteristic equals  $Z_Y = \overline{Y}/\hat{Y}$ , where  $\overline{Y}$  and  $\hat{Y}$  — mathematical expectation and standard of the stability reserve function (8).

For the random process model the probability of element failure during the term t is determined as follows:

$$Q(t) = \frac{\omega_q \cdot f(Z_Y) \cdot t}{\beta_w \sqrt{2 \cdot \pi}},\tag{10}$$

where 
$$\omega_q = \frac{\omega_c \cdot K_{tr1}}{\sqrt{1 + p_1^2}}$$
,  $t = T \cdot t_W$ ,  $\beta_W = \frac{\beta \sqrt{1 + k^2}}{k}$ ,  $k = \frac{1}{p_1}$ ,  $\omega_q$  — summary effective

frequency of RP  $\tilde{Y}(t)$ ;  $\omega_c$  — effective frequency of snow  $R_P$ ;  $K_{tr1}$  — trend coefficient accounts the snow load change during the year;  $f(Z_Y)$  — density ordinate distribution of  $R_P$   $\tilde{Y}(t)$  corresponding to the safety characteristic; T — service term in years;  $t_W$  — period of the stable snow load per year;  $\beta_W$  — coefficient of RP  $\tilde{Y}(t)$  structural complexity;  $\beta = 3$  — the same coefficient of snow RP.

The worked out method was allowed to evaluate the failure probability for beam—column elements of trusses. The probabilistic analysis of the bending tensile elements has analogy to the beam—column estimation. In this case the carrying capacity reserve function is:

$$\widetilde{Y}(t) = \widetilde{\sigma}_{y} - \frac{\widetilde{N}(t)}{A} - \frac{\widetilde{M}(t)}{W}, \tag{11}$$

where A – the section area; W – the section moment.

The partial derivatives of the function (11) are:  $j_1=1$ ;  $j_2=1/A$ ;  $j_3=-1/W$ .

The probabilistic computer analysis of the steel truss elements was realised. As a result the estimation of the roof structure probability was obtained.

## 5 CONCLUSIONS

The worked out method gives possibility to estimate reliability of a wide range of steel tubular trusses and to obtain the following results:

- a) all the tubular trusses have a sufficient reliability;
- b) reliability of beam—column truss elements is being reduced if the part of snow load is increasing in a general force;
- c) the lower chord is considerably more reliable than the top chord of tubular trusses;
- d) the lace bars of the analysed trusses are manufactured using two type of steel with the limit of yield difference in 1, 5 times; if is inadmissible because the reliability of beam—column diagonal bars is being considerably reduced.

### 6 REFERENCES

- [1] Bolotin V. V. Statistic Methods in Structural Mechanic. San Francisco: Holden Day, 1969.
- [2] Pashinski V. A. Building Structures under Atmospheric Loads in Ukraine. Kiev, UkrNII Proektstalconstructsia, 1999 (in Ukraine).
- [3] Pichugin S., Kharchenko Y. Probabilistic Analysis of Steel Beam Column Elements (SF 95) // Modern building materials, structures and techniques, Vilnius, Lithuania, II tom, vol. II, Vilnius, Technika, 1995. P. 145 150 (in Russian).
- [4] Pichugin S. F., Kharchenko Y. A. Probabilistic Estimation of Stability Reserve of Beam—Column Bars // Ways of Development of Building Efficiency: Collected volume. Kiev, ISIO, 1993. –P. 72 80 (in Russian).

# PROBABILISTIC ANALYSIS OF REDUNDANT STEEL STRUCTURES

## 1 INTRODUCTION

The assessment of a frame structural safety is rather a difficult task. In spite of many studies which have been done the problem of reliability analysis of redundant structures has not been solved yet. The principal ideas of applied probabilistic method were developed in works [1, 2]. The steel cross-frame which are treated as ductile systems are under consideration.

## 2 FAILURE MODE APPROACH

The nature of a system behavior in which elastic-plastic elements work in accordance with Prandtl's diagram, is under discussion. This type of behavior will be treated as a plastic failure mode. Structural systems of bending elements are of the primary interest including the redundant beams and frames. Their failure process is characterized by states (systems) which differ in the number of plastic hinges in critical sections. The system works with a full amount of connections (initial system  $C_0$ ) until the moment in the most loaded section reaches the limit value  $M_{Iy}$ :

$$M_{10} = \alpha_{10} \cdot F_0 = M_1^y = \mu_1 \cdot M_0^y, \qquad (1)$$

where:  $\alpha_{I0}$  — the influence factor for the 1-st section in  $C_0$  system;  $F_0$  — external load parameter;  $\mu_I$  — vector component of limit moments ratio in critical sections;  $M_0^y$  — system limit moment parameter.

The transmission from the original  $C_0$  to the first system  $C_1$  and the first limit load  $F_0^I = \mu_1 \cdot M_0^y / \alpha_{10}$  corresponds to this ratio (1). In the rest sections the dependencies M(F) are changing under further increase of the load (Fig. 1-a):

$$M_{21} = \alpha_{21}F_0 - \beta_{21}\mu_1 M_0^y; M_{31} = \alpha_{31}F_0 - \beta_{31}\mu_1 M_0^y,$$
 (2)

where  $\beta_{21} = (\alpha_{21} - \alpha_{20})/\alpha_{10}$ ,  $\beta_{31} = (\alpha_{31} - \alpha_{30})/\alpha_{10}$  — the factors of influence of a limit moment on the 1-st system moments introduced by N. S. Streletski [3].

The new graph segments have the same bending angles as in the case of the brittle failure mode but they are thrown to the segments  $M_{21}^w = \beta_{21}\mu_1 M_0^y$  and  $M_{31}^w = \beta_{31}\mu_1 M_0^y$ . As a result of equating  $M_{21} = \mu_2 M_0^y$  the transmission to the second system will be done and the following expression will be got for the second limit load:

$$F_0^{II} = M_0^y (\mu_2 + \mu_1 \beta_{21}) / \alpha_{21}. \tag{3}$$

By analogy the formula was obtained for the moments in the sections of the second system

$$M_{32} = \alpha_{32} F_0 - M_0^y [\mu_2 \beta_{32} + \mu_1 (\beta_{32} \beta_{21} + \beta_{31})]. \tag{4}$$

For the third limit load the expression will be as follows:

$$F^{III} = M_0^{y} [\mu_3 + \mu_3 \beta_{32} + \mu_1 (\beta_{32} \beta_{21} + \beta_{31})] / \alpha_{32}.$$
 (5)

It is possible to turn to the 3-rd, 4-th and other systems. In order to generalize the derived expression "i" will be marked as a number of a critical section; j—1,2 ..., "n" marks the order of examined system.

Then the general case will be as follows:

a) for the moments in i-section in n-system:

$$M_{in} = \alpha_{in} F_0 - M_0^y \sum_{k=1}^n \left[ \mu_k \left( \sum_{j=k+1}^n \beta_{ij} \prod_{k=1}^{j-1} \beta_{k+1,k} + \beta_{ik} \right) \right], \tag{6}$$

b) in order to simplify the received formulae for the practical use will mark the expression of a sum in the second term (6) as Bin and as a result the following expression will be derived:

$$M_{in} = \alpha_{in} F_0 - B_{in} M_0^y, (7)$$

c) for the influence factor of limit moment effect on the moment in i-section of j-system:

$$\beta = (\alpha_{ij} - \alpha_{i,j-1})/\alpha_{i,j-1}, \tag{8}$$

d) for the limit load in n-system:

$$F_0^{(n+1)} = M_0^y(\mu_{n+1} + B_{in})/\alpha_{in}.$$
(9)

The probabilistic estimation parameters of elements at any stage (in any failure state of a system) have been obtained in the space of bending moments on the base of received data.

The margin function  $\widetilde{Y}_{in}$ , its mathematics expectation  $\widetilde{Y}_{in}$  and the standard  $\widetilde{Y}_{in}$  have been determined as well:

$$\widetilde{Y}_{in} = \widetilde{M}_{n}^{y} - \widetilde{M}_{in}; \ \overline{Y}_{in} = \overline{M}_{0}^{y} (\mu_{n} + B_{in}) - \alpha_{in} \overline{F}_{0};$$

$$\widetilde{Y}_{in} = \left[ \left( \overline{M}_{0}^{y} \right)^{2} (\mu_{n} + B_{in})^{2} - \alpha_{in}^{2} \hat{F}_{0}^{2} \right]^{1/2}.$$
(10)

In this case the close correlation of limit moments in different system sections is supposed.

As shown in the figure 1-a, in sections the moment probabilistic distribution changes in different systems and affect differently the margin parameters (10). There is a  $f(M_{30})$  distribution in the original system, and usually it is the most compact one. After reaching the first limit load  $F_0^I$  the graph angle inclination increases (sometimes it can decrease). As a result the wider (narrower) distribution  $f(M_{si})$  is obtained on the base of the ratio (2). After the second level of the load  $F_0^I$  (3) had been overcome the distribution  $f(M_{32})$  was derived on the basis of equation (4).

Thus in different states different parts of various distributions should be taken into account (they are locked with dowels in Fig. 1-a). The obtained ratio was applied for the probabilistic computation of the redundant structures. It confirmed the important peculiarity of a random failure process of elastic-plastic redundant structures.

The idea is that the failure probability in all possible ways to the failure mechanism becomes closer when the next plastic hinges occur. This peculiarity is the probabilistic reflection of a fundamental character of elastic-plastic systems. It means that the value of a limit load for the examined failure mechanism does not depend on order of system failure connections [3]. But there are different ways of failure for the same failure limit loads. It is shown in Fig.1-b for the fixed beam. The first way I is characterized by the passing through the most loaded sections (the failure formula  $C_0$ - $C_3$ - $C_2$ - $C_1$  or simply (0321) and its graph suits to a limit  $M_v$  and limit load  $F_{III}$  down. First the ways II (0123) and IV (0213) pass through the less loaded  $M_y$  section and the  $F_{III}$  one. Their graphs pass above the limit values  $M_{\nu}$  and  $F_{III}$  and then return to them. The last segments of all graphs could be different but all of them give the same limit load  $F_{III}$  for  $M_{\nu}$  ordinate, i.e. for the general system failure (Fig. 1-b). These ways of failure can not be taken into account by applying the deterministic computation as they pass above the limit level  $M_{\nu}$ . Such ways will be called "overlimited", for example the II,IV,V ways. The probabilistic estimation takes into consideration the statistic changeability and independence of load and as well as the strength of different sections. As the result of this all ways of system failure are possible with different probability of realization. For all that, it is obvious the overlimited ways should not be neglected as they fall a little outside the limit level.

The beam (Fig. 1-b) is the model of one possible failure mechanism all approaching ways to it give the same possibility of a system failure. As all these ways are closely correlated any possible failure way should be taken into account in this case (so called the weak member concept). It is recommended to examine the ways which are approaching down the limit parameters. These ways correspond to the deterministic method. These ways, for example I-0321 (Fig. 1-b), demonstrate the benefit of the whole system work before the system failure occurred as against the work before the first plastic hinge.

## 3 PROBABILISTIC METHOD OF LIMIT EQUILIBRIUM

The chosen failure mechanism has an invariant limit load and the probability of a realization. Therefore only the last system can be observed, i.e. it is the failure mechanism that should be examined but the observation of numerous intermediate systems can be neglected.

The approach like this has been realized in the form of following method [4]. Suppose that the n-redundant frame is under consideration. The n+1 linear equation system is simulated for this. It is as follows:

$$\sum_{i=1}^{n} M_{ij} X_{j} + F_{0} M_{i0} = M_{i}^{y}, \qquad (11)$$

where:  $M_{ij}$  — moments in i-section of the basic system caused by the add unknown  $X_j = 1$ ;  $M_{i0}$  — the same moments caused by the external loads the parameter of which is taken as F0=1;  $M_i^y$  — the limit moment load in i-section.

The combined theorem of a limit equilibrium method is used. So while solving the system equation the position of plastic hinges included into the chosen failure mechanism is taken into account. While realizing this mechanism the mathematics expectation  $\overline{F}_0$  of a limit load frame parameter is defined as follows:

$$\overline{F}_0 = \sum_{\nu=1}^{n+1} \frac{A_{\nu,n+1}}{D} \,\mu_{\nu} \overline{M}_0^{\nu} \,, \tag{12}$$

where:  $\mu_{\nu}$  — components of a vector of limit moments ratios in plastic hinges; D — equation system determinant;  $M_0^{\nu}$  — mean value of a frame limit moment parameter;  $A_{\nu,n+1}$  — algebraic compliments of elements  $M_{\nu 0}$  of D determinant.

Suppose that the strengths in different sections are independent, then the standard deviation of a limit load parameter  $F_0$  is calculated as follows:

$$\hat{F}_0 = \left[ \sum_{\nu=1}^{n+1} \left( \frac{A_{\nu,n+1}}{D} \mu_{\nu} \right)^2 \right]^{\frac{1}{2}} \hat{M}_0^{\nu}.$$
 (13)

In general (12) and (13) formulae connect the numerical characteristics of a frame random strength with the ones of the isolated elements.

Then in the space of load parameter the characteristics of margin function of frame load carrying capacity are determined. It allows to estimate the frame failure probability:

$$\overline{Y} = \overline{F}_0 - \overline{Q} ; \ \hat{Y} = (\hat{F}_0^2 + \hat{Q}^2)^{1/2},$$
 (14)

where:  $\overline{Q}$  and  $\hat{Q}$  — both mathematical expectation and standard deviation of the external load parameter.

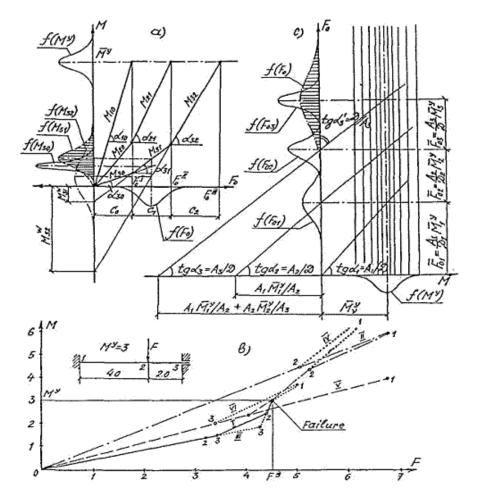


Fig. 1. Redundant structure plastic model of failure: a) model probabilistic analysis; b) the comparison of different failure models; the geometrical interpretation of formulae (17)-(18).

This method is usually applied to the major (most probable) mechanism of a frame failure. As a result of which it gives the low estimation of a system failure probability which coincide with the full estimation in the most real cases.

## 4 THE COMPARISON OF SUGGESTED METHODS

The computation results on formula (14) and formula (10) of failure modes coincide. It can be proved in the case of transition from the space load parameter to the space of the bending moment. Then the moment in the last system is derived from equation (12) which equals:

$$\overline{M}_{in} = \overline{F}_{in} \frac{D}{A_{n+1,n+1}} - \sum_{\nu=1}^{n} \frac{A_{\nu,n} \mu_{\nu}}{A_{\nu,n+1}} \overline{M}_{0}^{\nu} . \tag{15}$$

Let's mark

$$\alpha_{in} = \frac{D}{A_{n+1,n+1}}; \ B_{in} = \sum_{\nu=1}^{n} \frac{A_{\nu,n} \mu_{\nu}}{A_{\nu,n+1}}, \tag{16}$$

the formula (7) of the failure mode approach is obtained.

For the system of which the degree of static indeterminacy equals 2, the expression (12) is presented in the form of

$$\overline{F}_0 = \frac{A_1}{D} \overline{M}_1^y + \frac{A_2}{D} \overline{M}_2^y + \frac{A_3}{D} \overline{M}_3^y.$$
 (17)

In Fig. 1-c the combined formulae are illustrated by the straight lines with the angular coefficients  $tg\alpha_V = A_V/D$ . When they cross the vertical line  $M = \overline{M}_v^y$  these lines give the segments  $\overline{F}_{0v}$  which reflect every plastic hinge contribution into the mean limit load capacity. Let's dwell upon the formula for the moment in accordance with

$$M_{32} = \overline{F}_0 \frac{D}{A_3} - \frac{A_2}{A_3} \overline{M}_2^y + \frac{A_1}{A_3} \overline{M}_1^y.$$
 (18)

The 1-st straight line equation mentioned above was derived in M and F0 coordinates. It is an identical to the expression (4) with the angular coefficient  $tg\alpha_3 = \alpha_{32} = D/A_3$ . This line cuts the segment at the ordinate axis which equals to the sum of the rest members in the formula (18) (Fig 1-c). If the sequence of plastic hinges appearance is changed the absolute value D and  $A_{\nu}$  will be unchanged, but their product signs sequence in the expression will be changed (18). As a result of this equations of the last straight lines will be different while the values  $\overline{F}_0$  and  $\hat{F}_0$  from the formula (12-13) remain unchangeable.

Thus, two different approaches of frame probabilistic evaluation were under discussion. The failure mode method defines the strength in different system states, which are compared with the load carrying capacities of elements. The limit equilibrium method determines the whole system load carrying capacity which is compared with the external load. The obtained estimations of the frame probability coincide.

## 5 THE DEVELOPMENT OF PRACTICAL EVALUATION OF STEEL FRAME

In the existing steel frames all different failure modes are possible. But they have various possibilities of appearance. The probabilistic method of combined mechanisms describes the failure modes perfectly [5], It should be noted that all the rest methods mentioned above give sufficiently accurate estimation of steel frames failure probability.

In accordance with these methods different numerical computations of redundant frames and beams were done. It offers to introduce a new coefficient  $\gamma s > 1$ , which takes into account the specific character of work of failure of redundant structure

$$\gamma_s = M_0^y(1) / M_0^y(S_i), \tag{19}$$

where:  $M_0^y(1)$  — limit moment of redundant structure parameter; the system works up to the first frame failure;  $M_0^y(S_j)$  — limit moment parameter, which corresponds to the majorfailure mechanism.

Formulae parameters (19), the condition of failure probability equality of isolated elements Q(1) and  $Q(S_j)$  of the whole redundant structure are defined. The smallest parameter  $\gamma_s = 1.14$  was determined for G-frame which is destroyed in accordance with a partial mechanism with 2 plastic hinges. For the cases of partial beam failure mechanism of redundant structures increases up to 1.19-1.25. If the degree of static indeterminacy increases and realizes full schemes of failure with great number of plastic hinges the coefficient y, can be increased up to 1.27-1.35.

On the base of obtained data the design coefficient  $\gamma_s = 1.10$  is proposed. This value is calculated with the reserve of safety in rough and could be defined more precisely later. The proposed coefficient is recommended for the evaluation of solid bending steel elements of redundant structures. It is supposed that the plastic equalization of moments and the optimization of redundant structures are not realized.

## **6 CONCLUSIONS**

For ductile steel redundant structures the probabilistic estimation parameters of elements at any stage have been obtained. The margin function of frame load carrying capacity has been determined as well. The numerical computations offer to introduce a design coefficient which takes into account the specific character of work and failure of redundant structures.

### 7 REFERENCES

- [1] Pichugin S., Relialibity Design Method of Stell Structures. XL Konferencja Naukowa KILiW PAN i Komitetu Nauki PZITB. Rzeszow-Krynica-Warszawa, 1994. p. 133-140 (in Russian).
- [2] Pichugin S., Reliability Estimation of Steel Elements Under Variable Loads. XLI Konferencja Naukowa KILiW PAN i Komitetu Nauki PZITB. Krynica, 1995. p. 151-156.
- [3] Streletski N. S., Analysis of Failure Process of Elastic-Plastic Systems //Engineering Structures: Collected Book of MICI-N5-M., 1947.-p.68-l 15 (in Russian).
- [4] Pichugin S. F., Semko A. V., On One Estimation of Elastic-Plastic Redundant Frame Reliability / Different Educational Institutions Collected Book, Kuybyshev, 1990. p. 124-128 (in Russian).
- [5] Pichugin S. F., Chihulin V. P., Steel Frame Reliability Estimation Based on Limit Balance Approach // Effective Building Materials and Structures Used for Building and Structures Amounting/ Collected Book of Scientific Works. K.:UMK VO, 1992.- p.84-90 (in Russian).