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Compatibility on naturally ordered endotopism semigroups of a partial equivalence

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An ordered pair (φ, ψ) of transformations φ and ψ of a nonempty set X is called an *endotopism* [1] of a relation $\rho \subseteq X \times X$ if for all $a, b \in X$ the condition $(a, b) \in \rho$ implies $(a\varphi, b\psi) \in \rho$. The set of all endotopisms of ρ is a semigroup with respect to the componentwise multiplication operation. This semigroup is called the *endotopism semigroup* of ρ and it is denoted by $Et(X, \rho)$.

For an arbitrary semigroup S the binary relation $\leq$ defined by

$$a \leq b \text{ iff } a = xb = by, a = xa \text{ for some } x, y \in S^1$$

is a partial order on S which called the *natural partial order* [2]. Here S^1 denotes S if the given semigroup has an identity or, otherwise, S with an adjoined identity.

An element c of a semigroup S is said to be *left (right) compatible with the natural partial order* $\leq$ on S if for all $a, b \in S$ the condition $a \leq b$ implies $ca \leq cb$ $(ac \leq bc)$.

A binary relation on some set is called a *partial equivalence* [3] if it is symmetric and transitive. In [4] necessary and sufficient conditions for elements of the naturally ordered endomorphism semigroup of an equivalence to be left or right compatible were obtained. We consider the same problem for elements of the naturally ordered endotopism semigroup of a partial equivalence.

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